Optical Spin-to-Orbital Angular Momentum Conservation in Inhomogeneous Anisotropic Media

Cha Seoncheol

Department of Physics and Interdisciplinary Program of Integrated Biotechnology, Sogang University

Soft-Matter Optical Spectroscopy

SOGANG UNIVERSITY

Contents

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Optical Spin-to-Orbital Angular Momentum Conversion in Inhomogeneous Anisotropic Media

L. Marrucci,* C. Manzo, and D. Paparo

Dipartimento di Scienze Fisiche, Università di Napoli "Federico II" and CNR-INFM Coherentia
Complesso di Monte S. Angelo, via Cintia, 80126 Napoli, Italy

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We demonstrate experimentally an optical process in which the spin angular momentum carried by a circularly polarized light beam is converted into orbital angular momentum, leading to the generation of helical modes with a wave-front helicity controlled by the input polarization. This phenomenon requires the interaction of light with matter that is both optically inhomogeneous and anisotropic. The underlying physics is also associated with the so-called Pancharatnam-Berry geometrical phases involved in any inhomogeneous transformation of the optical polarization.

"Angular Momentum in light"

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A monochromatic light beam traveling along a given axis z can transport angular momentum oriented as the propagation direction in two different forms.

Spin form PR 50 115 (1936)

Orbital form PRA 45 8185 (1992)

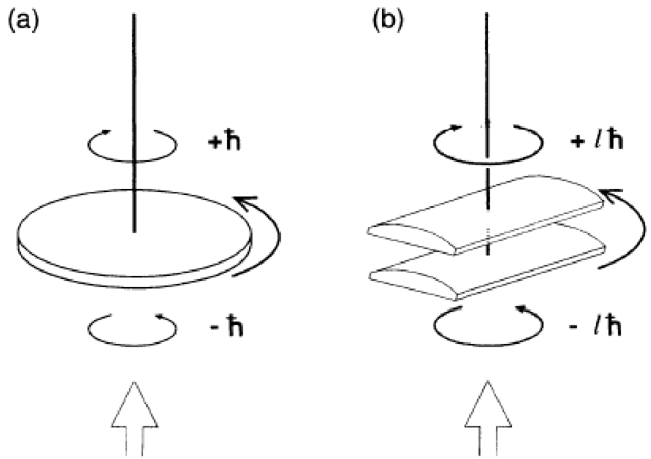


FIG. 1. (a) A suspended $\lambda/2$ birefringent plate undergoes torque in transforming right-handed into left-handed circularly polarized light. (b) Suspended cylindrical lenses undergo torque in transforming a Laguerre-Gaussian mode of orbital angular momentum $-l\hbar$ per photon, into one with $+l\hbar$ per photon.

Spin form PR 50 115 (1936)

+ $\hbar$: left-handed circularly polarized light

- $\hbar$: right-handed circularly polarized light

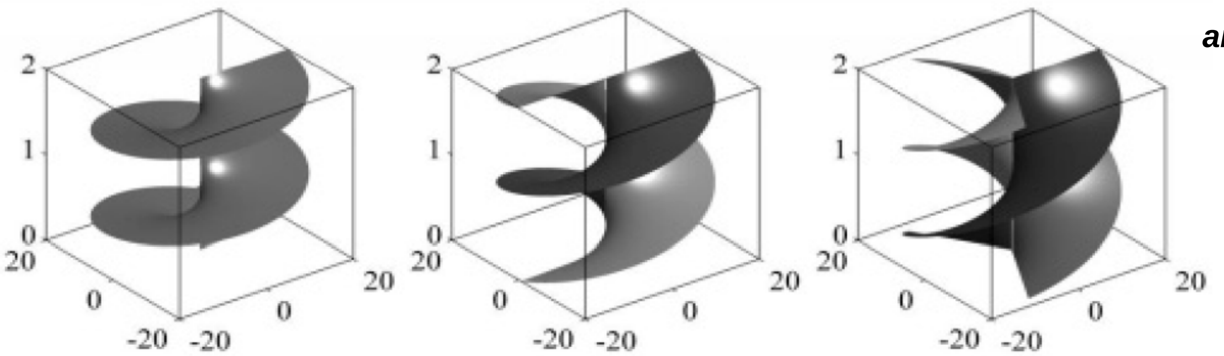
Orbital form PRA 45 8185 (1992)

Laguerre-Gaussian modes : Laguerre-Gaussian modes have rotational symmetry along their propagation axis and carry an intrinsic rotational orbital angular momentum $n\hbar$.

The number n defines by how many full cycles of 2π the phase is changing when one goes around the axis of propagation ρ=0, and thus defines the chirality of the mode.

$$E^L_{m,n} = \frac{e^{-im\phi}}{w(\zeta)}\left(\frac{\rho}{w(\zeta)}\right)^n L^n_m\left(\frac{2\rho^2}{w^2(\zeta)}\right)\exp\left[ikz - \frac{\rho^2}{w_0^2(1+i\zeta)} - i\psi^L_{n,m}\right]$$

arXiv:physics/0410021



m = 1 m = 2 m = 3

"Helical Mode"

The electric field of a beam carrying a well defined value of the orbital angular momentum can be written as $\vec{E}(r,\varphi) = E_0(r)\exp(im\varphi)$

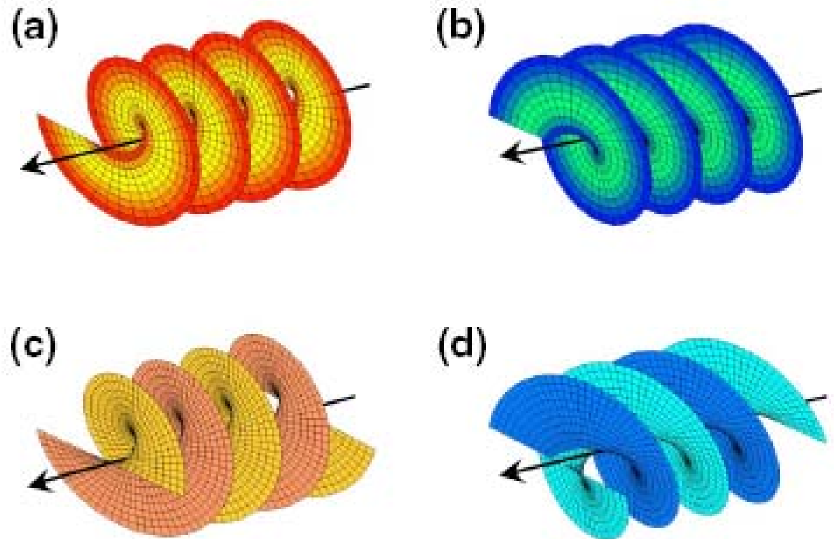


FIG. 1 (color online). Examples of helical waves. Represented are the wave fronts of helical modes for helicities (a) $m = +1$, (b) $m = -1$, (c) $m = +2$, and (d) $m = -2$.

Light propagates in

1. homogeneous and isotropic transparent medium :

spin / orbital angular momentum are separately conserved.

2. Inhomogeneous and anisotropic medium :

spin / orbital angular momentum have simultaneous coupling with matter PRE 69 056613 (2004)

Condition :

Spin / orbital angular momentum exchange that remain

always exactly opposite to each other

The exchange with matter :

spin angular momentum affects the direction of the

exchange of the orbital angular momentum

Consider planar slab that

Across : homogeneous birefringent phase retardation π

Parallel : inhomogeneous orientation of the fast optical axis

Wave plate

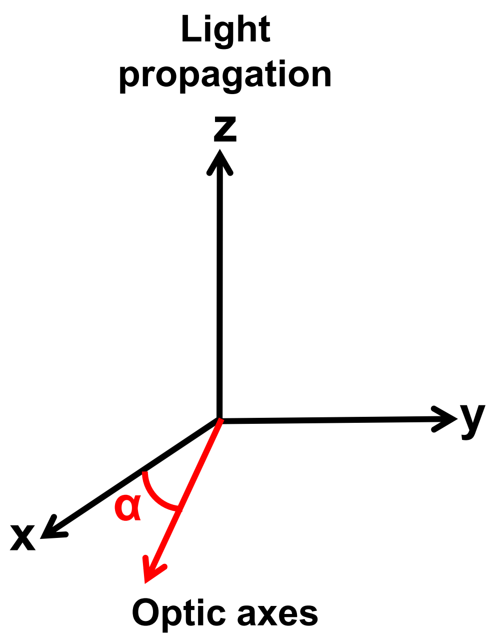
$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

Taking φ=π (half-wave plate)

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Space-dependent matrix is

$$R(-\alpha)\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot R(\alpha) = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$



Spin-to-angular momentum

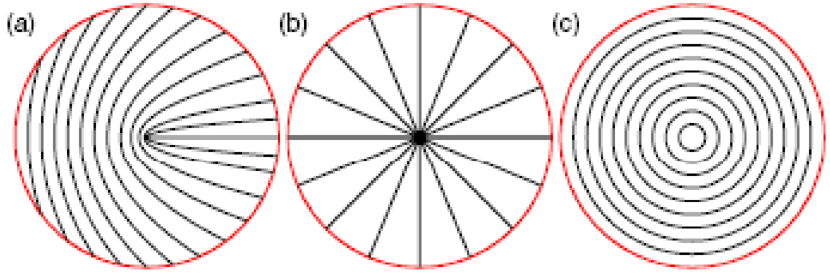


FIG. 2 (color online). Examples of q plates. The tangent to the lines shown indicates the local direction of the optical axis. (a) $q = 1/2$ and $\alpha_0 = 0$ (a nonzero α_0 is here just equivalent to an overall rigid rotation), which generates helical modes with $m = \pm 1$; (b) $q = 1$ with $\alpha_0 = 0$ and (c) with $\alpha_0 = \pi/2$, which can both be used to generate modes with $m = \pm 2$. The last two cases correspond to rotationally symmetric plates, giving rise to perfect spin-to-orbital angular momentum conversion, with no angular momentum transfer to the plate.

Opt.Comm. 251, 306 (2005)

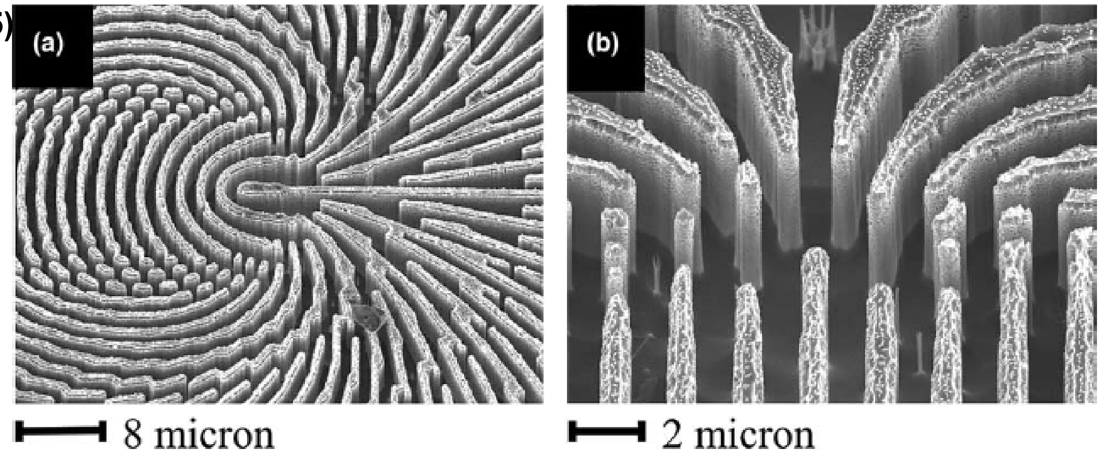


Fig. 4. Scanning electron microscope images of the central parts of the elements having topological charge (a) $l = 3$, and (b) $l = 4$.

waveplate $$\vec{E}_{out} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} E_0 \begin{pmatrix} 1 \\ i \end{pmatrix} = E_0 \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Left-circular pol. light **Right-circular pol. light**

Space-dependent matrix

Insert a left-circular polarized plane wave

$$\vec{E}_{out} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} E_0 \begin{pmatrix} 1 \\ i \end{pmatrix} = E_0 e^{i2\alpha} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

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Insert a Right-circular polarized plane wave

Soft-Matter Optical Spectroscopy

Input polarization of the light controls orbital helicity of output wave front.

: spin angular momentum transformed into orbital angular momentum

Insert a left-circular polarized plane wave

$$\vec{E}_{out} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} E_0 \begin{pmatrix} 1 \\ i \end{pmatrix} = E_0 e^{i2\alpha} \begin{pmatrix} 1 \\ -i \end{pmatrix} = E_0 e^{i2q\varphi} e^{i2\alpha_0} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Opposite sign

$$\vec{E}_{out} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} E_0 \begin{pmatrix} 1 \\ -i \end{pmatrix} = E_0 e^{-i2\alpha} \begin{pmatrix} 1 \\ i \end{pmatrix} = E_0 e^{-i2q\varphi} e^{-i2\alpha_0} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

Insert a Right-circular polarized plane wave

Spin-to-angular momentum

In previous page

Spin part : ±ħ
-> changing 2ħ
Orbital part : mħ
-> changing 2qħ
L-G Beam $e^{im\varphi}$

Consider

q=1 -> angular momentum changes ± 2h

-> convert to spin to orbital

q≠1 -> angular momentum changes ± 2h(q-1)

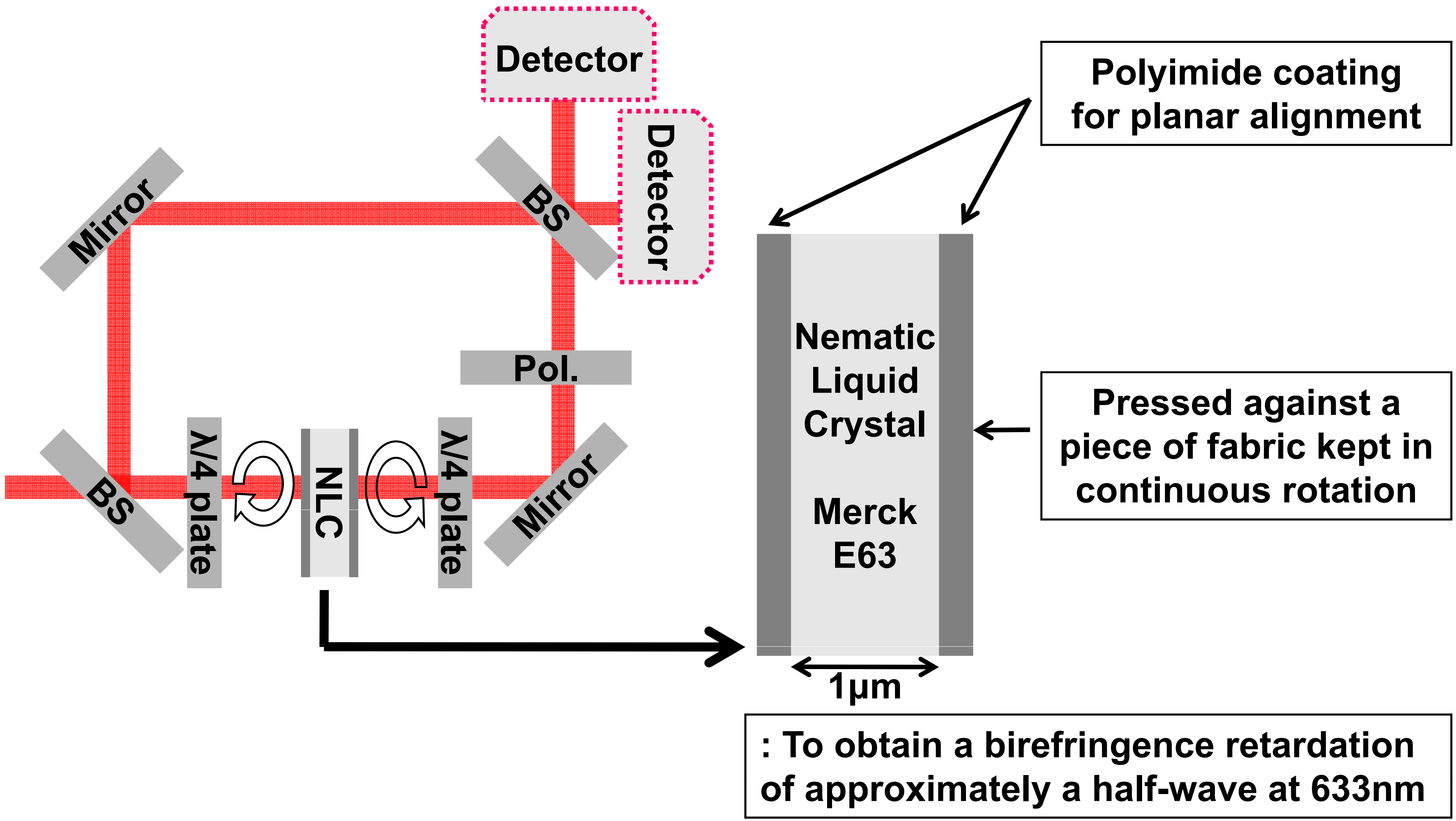
-> convert to spin to orbital
+
control the direction of the angular momentum exchange with plate

Gaussian Beam
(Lowest order L-G Beam)

L-G Beam

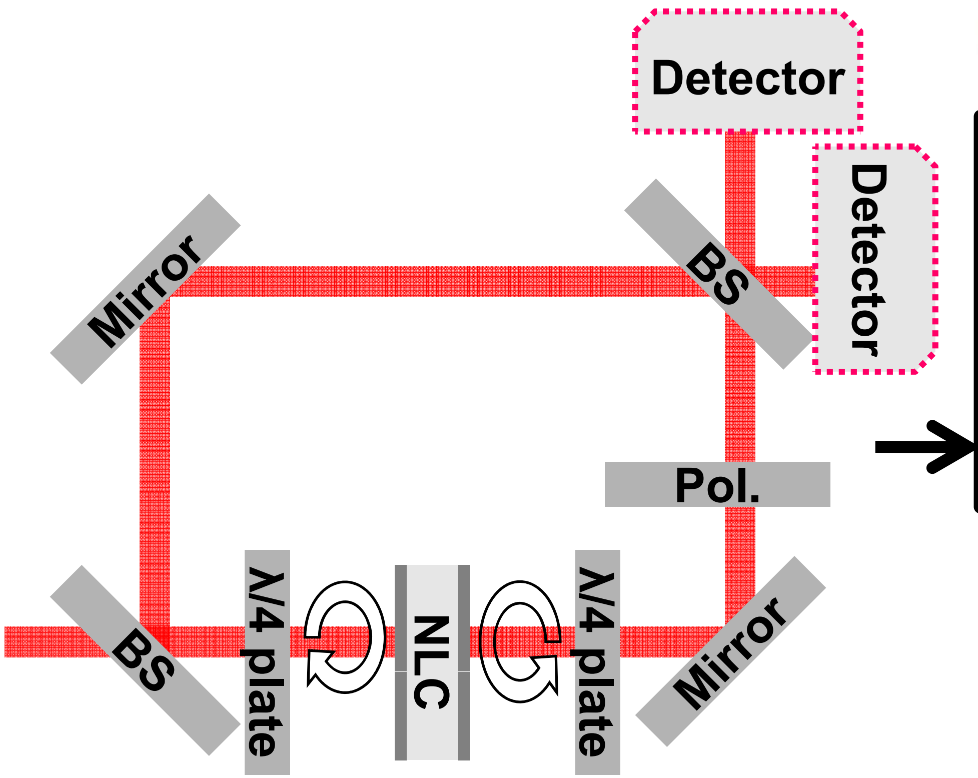
$$\vec{E}_{out} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} E_0 \begin{pmatrix} 1 \\ i \end{pmatrix} = E_0 e^{i2\alpha} \begin{pmatrix} 1 \\ -i \end{pmatrix} = E_0 e^{i2q\varphi} e^{i2\alpha_0} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Spin-to-angular momentum



APL 88 221102 (2006)

Spin-to-angular momentum



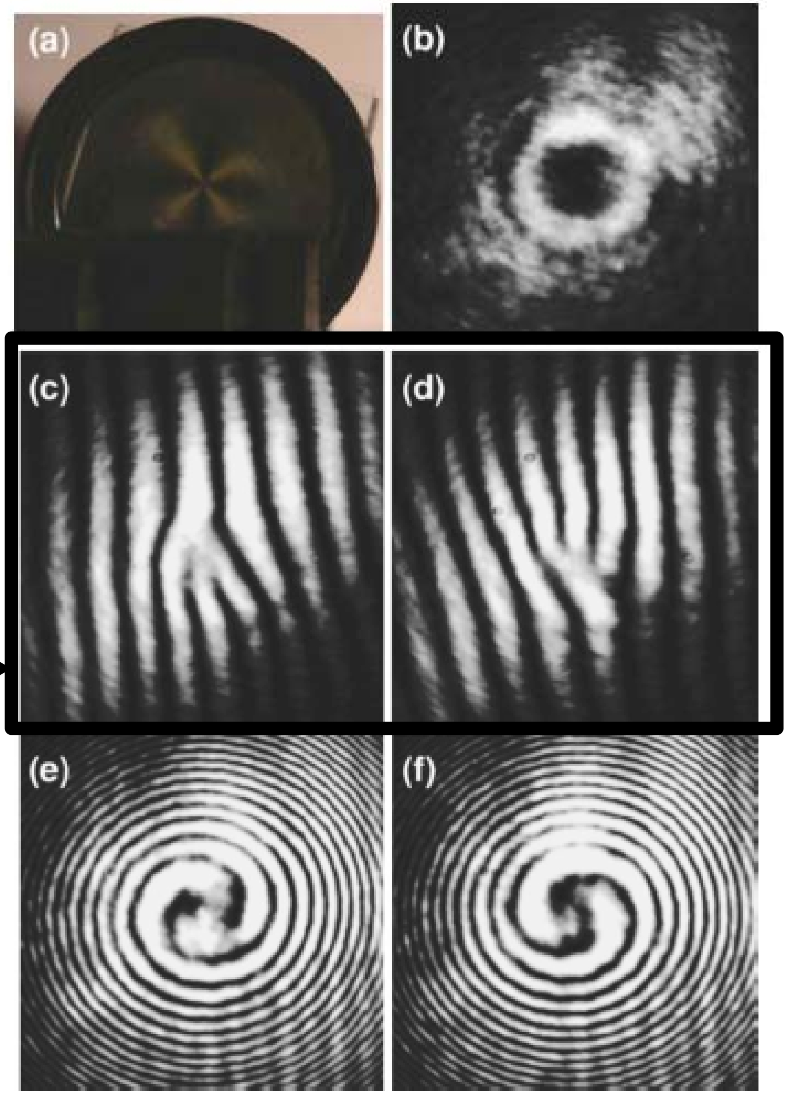
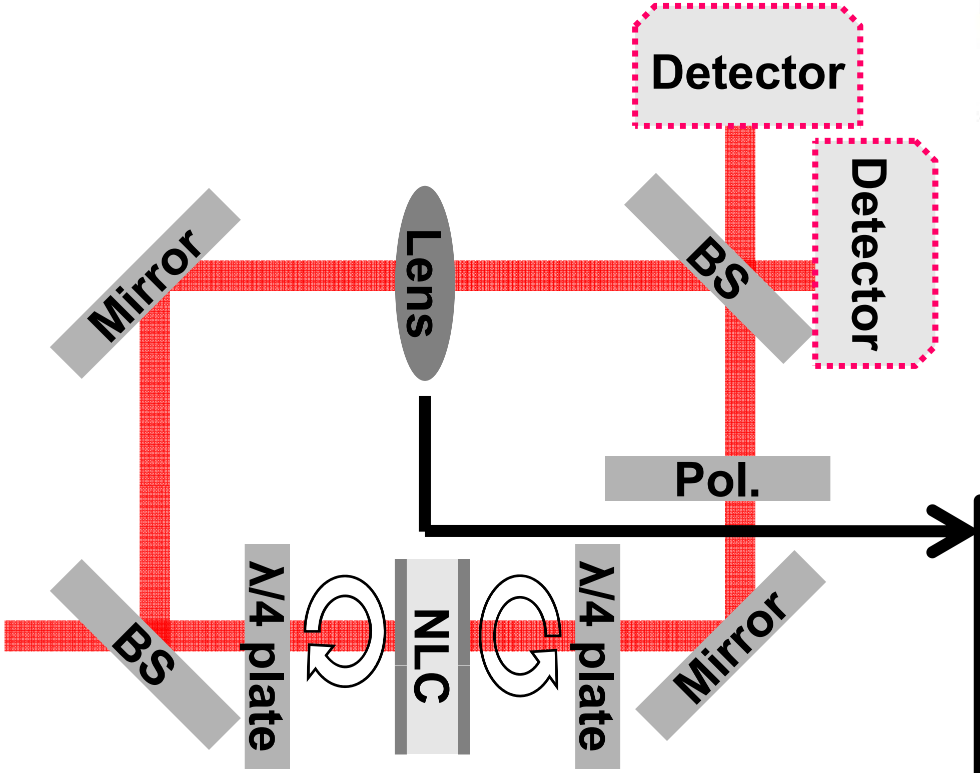
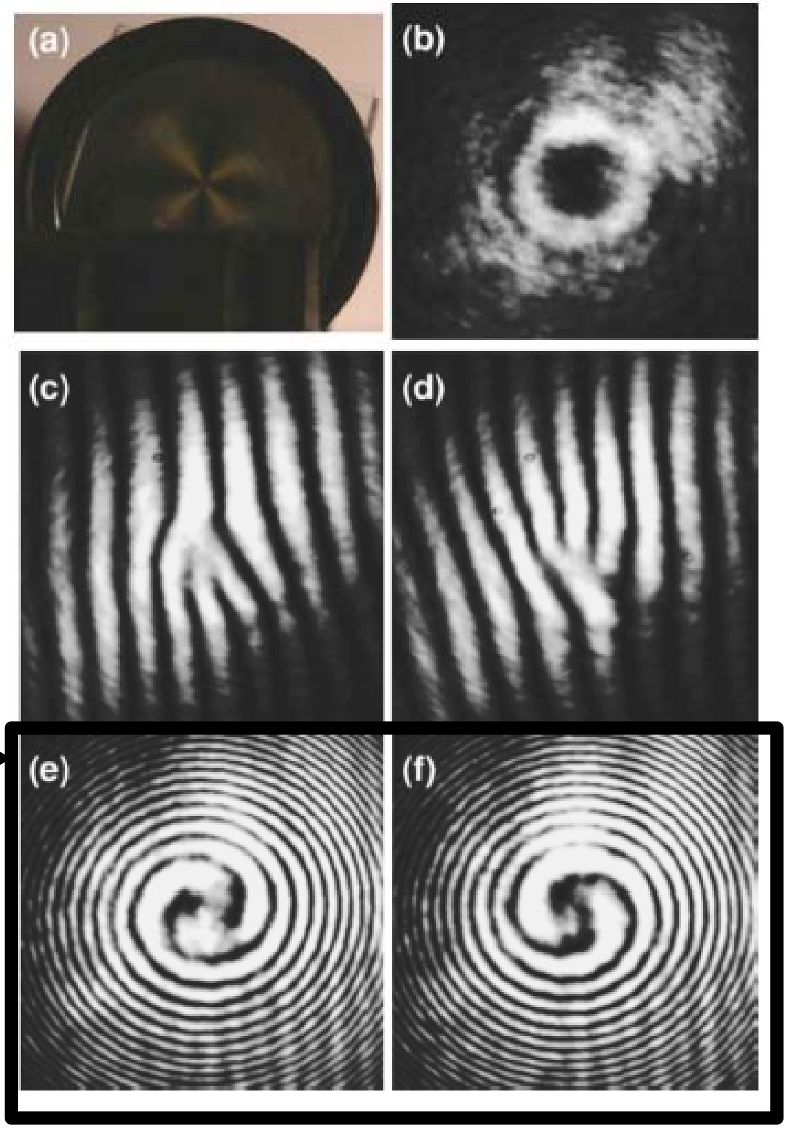


FIG. 2. Experimental images. (a) A LC q plate held between crossed polarizers, showing the expected pattern for $q=1$ geometry. (b) "Doughnut" intensity profile of the beam emerging from the q plate. (c)–(f) Interference patterns of helical modes generated by our q plate. (c) and (d) panels refer to the plane-wave reference geometry, (e)–(f) panels to the spherical-wave reference one. Panels on the left, (c) and (e), are for a left-circular input polarization and those on the right, (d) and (f), for a right-circular one.

APL 88 221102 (2006)

Spin-to-angular momentum





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Soft-Matter Optical Spectroscopy

Space-dependent matrix

Insert a left-circular polarized plane wave

$$\vec{E}_{out} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} E_0 \begin{pmatrix} 1 \\ i \end{pmatrix} = E_0 e^{i2\alpha} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

This phase originates from geometrical nature

: Pancharatnam-Berry Phase Opt. Lett. 27, 1875 (2002)

Pancharatnam' s Question :

"Given two beams of polarized light, is there a natural way to compare the phase of these beams?"

-> interference between two polarized beams as "in phase" when the resultant intensity is maximum

-> This provides a "connection" between any two states of polarization which are not orthogonal.

Pancharatnam Phase : $Pol_1 \rightarrow_{\text{in phase}} Pol_2 \rightarrow_{\text{in phase}} Pol_3 \rightarrow_{\text{in phase}} Pol_{1,out}$

1 and 3 are not necessarily in phase

Soft-Matter Optical Spectroscopy

Berry's Question : "What is the phase of the system?"

Time-independent Hamiltonian

$$H\psi_n(x) = E_n\psi_n(x)$$

$$\Psi_n(x,t) = \psi_n(x)e^{-\frac{iE_n t}{\hbar}}$$

Time-dependent Hamiltonian

$$H(t)\psi_n(x,t) = E_n(t)\psi_n(x,t)$$

$$\Psi_n(x,t) = \psi_n(x,t)e^{-\frac{1}{\hbar}\int_0^t E_n(t')dt'}e^{i\gamma_n(t)}$$

Dynamic Phase $\theta_n(t) \equiv -\frac{1}{\hbar}\int_0^t E_n(t')dt'$

: it generalizes the standard factor $(-E_n t/\hbar)$

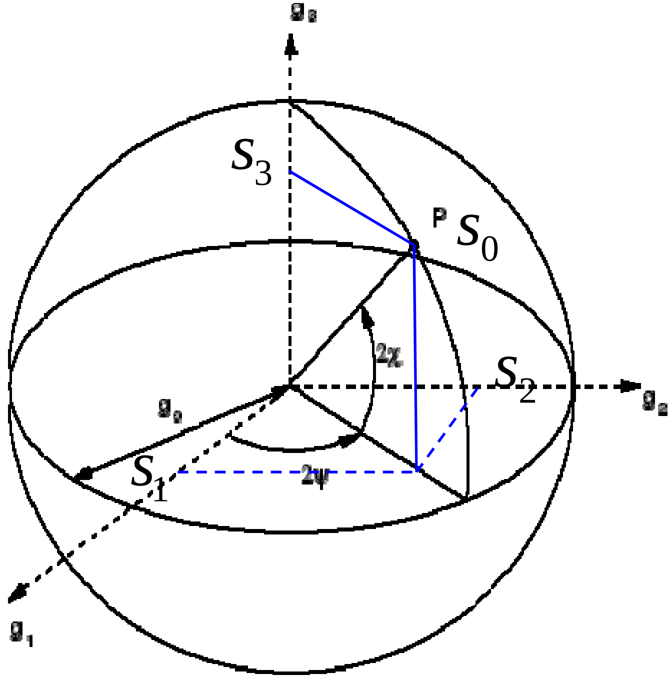
Geometric Phase

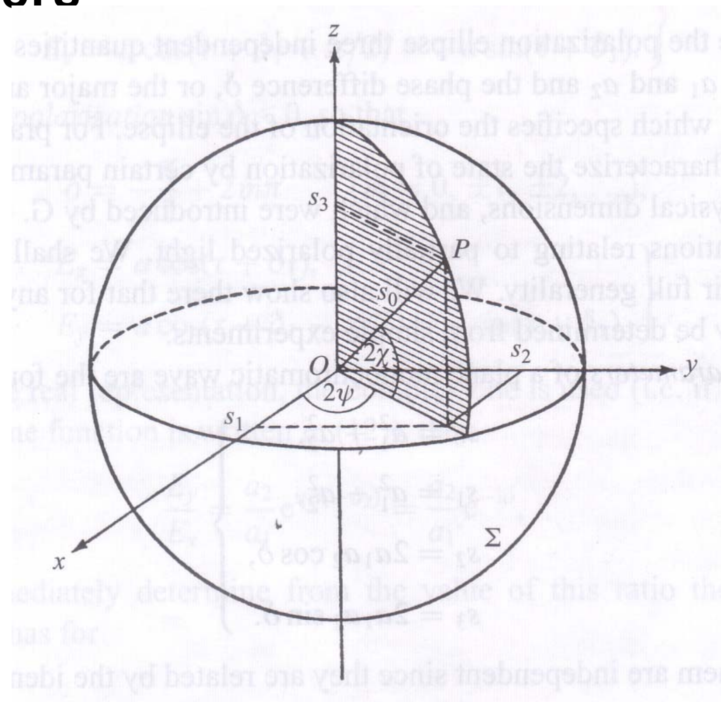
All we can say is that the adiabatic theorem does not rule out such a factor, since the particle is still "in the nth eigenstate" whatever the value. of γ_n.

Pancharatnam-Berry’ s Phase

:

Poincare Sphere & Stokes Parameters





$$s_1 = s_0 \cos 2\chi \cos 2\psi$$

$$s_2 = s_0 \cos 2\chi \sin 2\psi$$

$$s_3 = s_0 \sin 2\chi$$

$$s_0 = a_1^{\ 2} + a_2^{\ 2}$$

$$s_1 = a_1^{\ 2} - a_2^{\ 2}$$

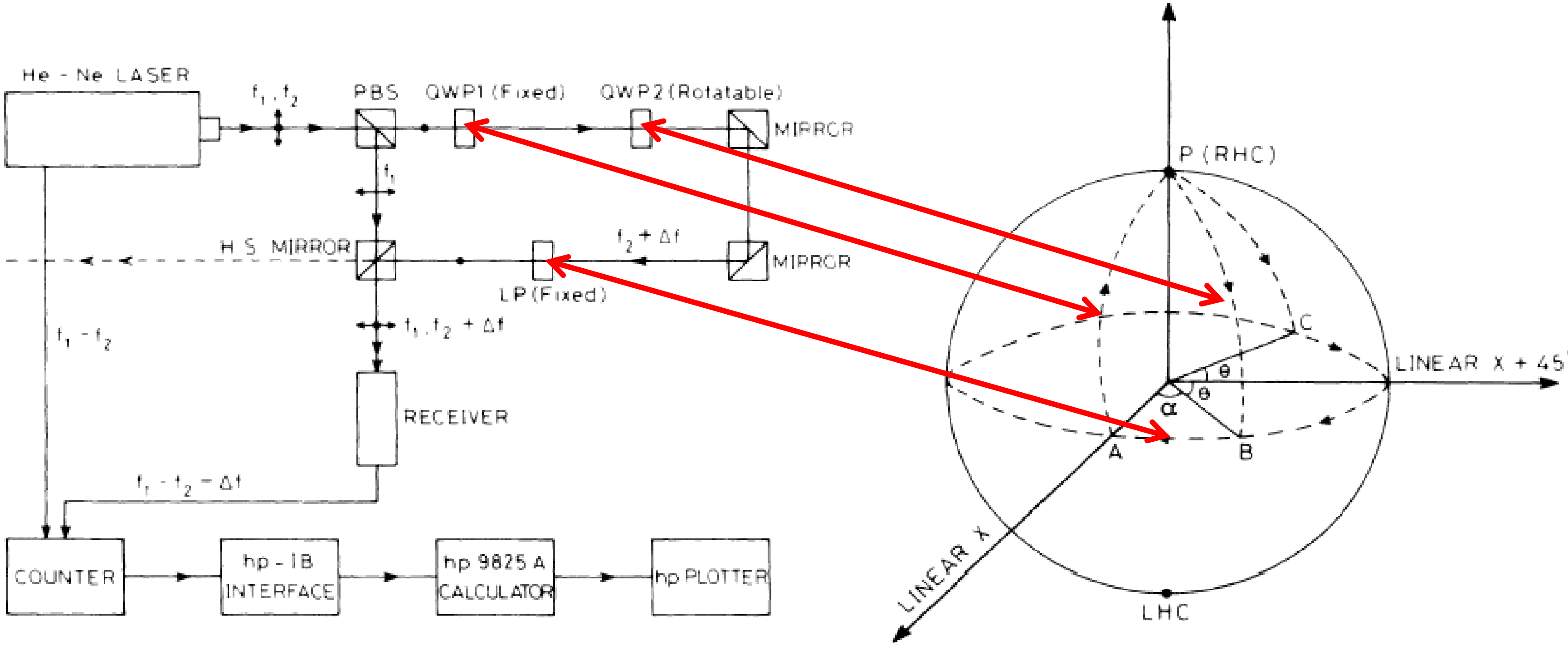
$$s_2 = 2a_1 a_2 \cos\delta$$

$$s_3 = 2a_1 a_2 \sin\delta$$

Pancharatnam Phase : $Pol_1 \rightarrow_{in\ phase} Pol_2 \rightarrow_{in\ phase} Pol_3 \rightarrow_{in\ phase} Pol_{1,out}$

1 and 3 are not necessarily in phase.

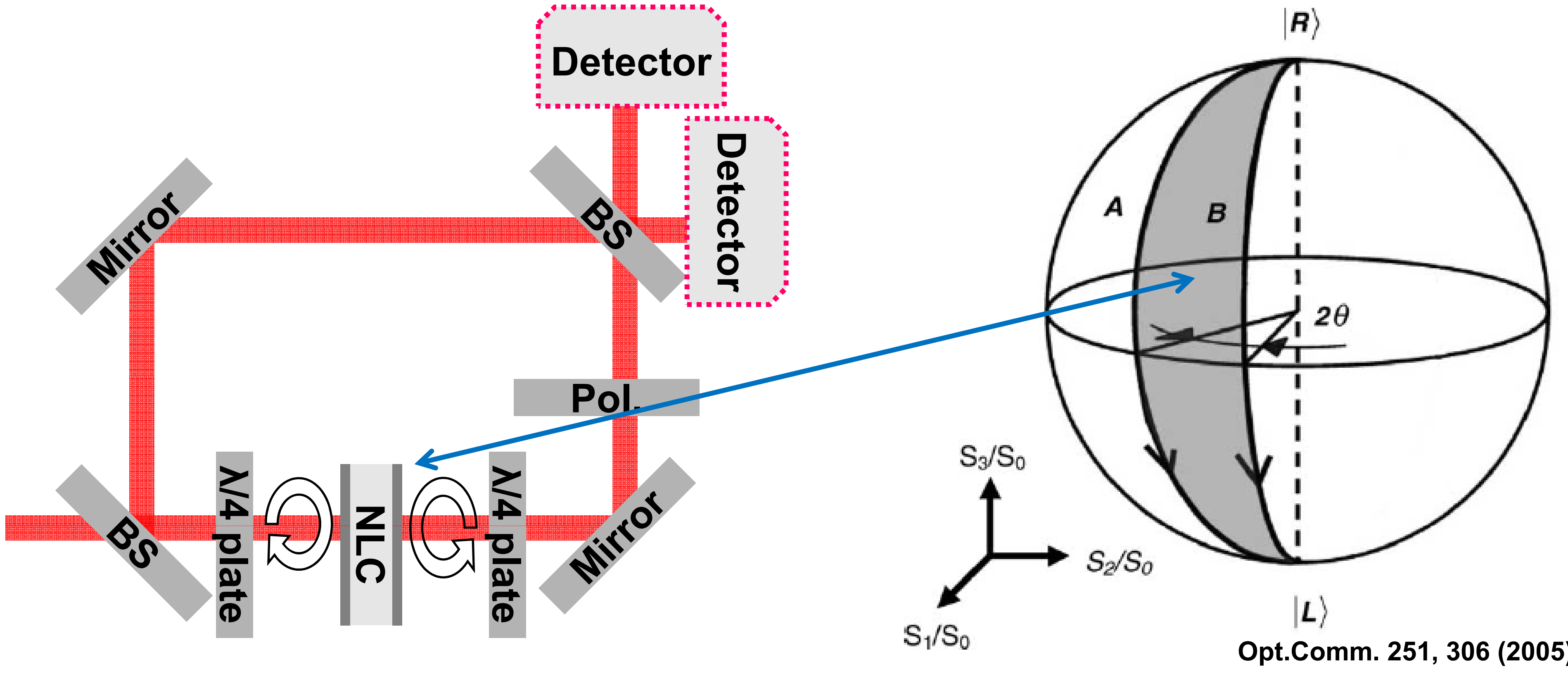
A Simple example of Pancharatnam Phase PRL 60 1211 (1988)



Pancharatnam Phase : $Pol_1 \rightarrow_{in\ phase} Pol_2 \rightarrow_{in\ phase} Pol_3 \rightarrow_{in\ phase} Pol_{1,out}$

1 and 3 are not necessarily in phase.

This Case



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