Prism coupling method for determining refractive index and thickness

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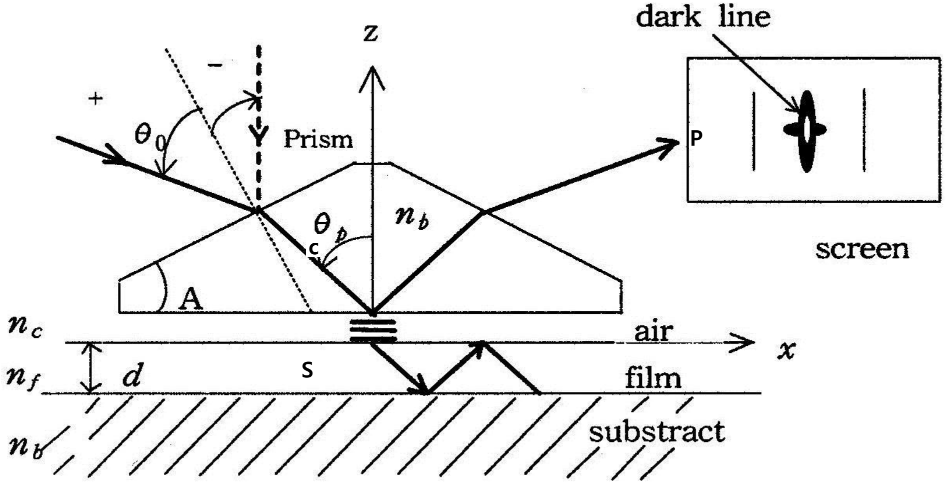
Introduction

problem

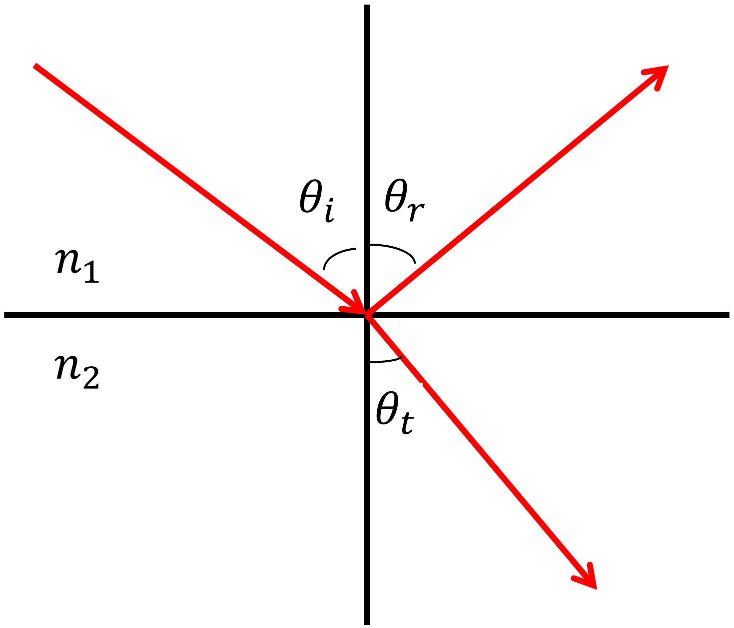
Unknown parameter : reflective index, thickness glass

Necessary Non Destructive method

Prism coupling

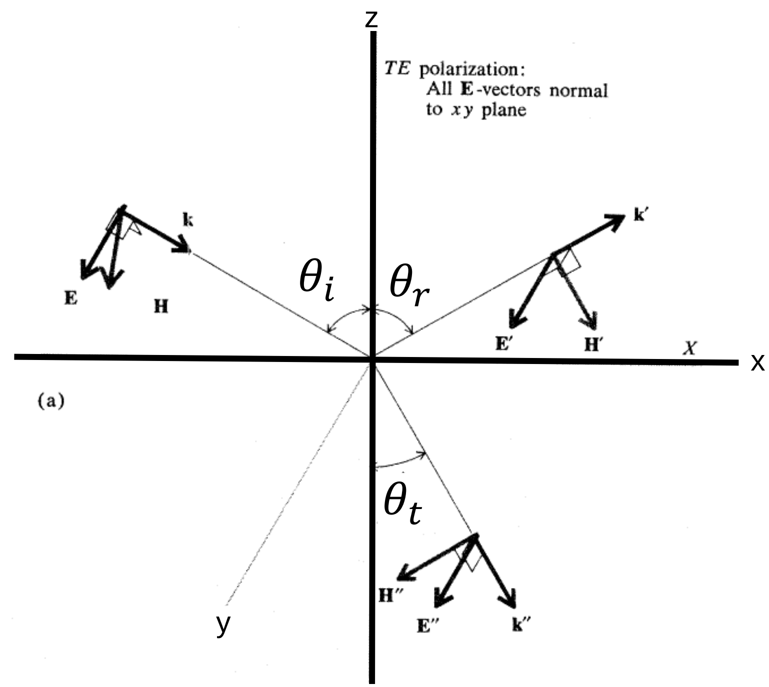


Fresnel equation



law of reflection $\theta_i = \theta_r$

snell's law $n_1 sin\theta_i = n_2 sin\theta_t$



Reflection coefficients are defined as amplitude ratios

$$r_s = \left[\frac{E'}{E}\right]_{TE} \qquad r_s = \left[\frac{E'}{E}\right]_{TM}$$

$$t_p = \left[\frac{E''}{E}\right]_{TE} \qquad t_p = \left[\frac{E''}{E}\right]_{TM}$$

Fresnel equation

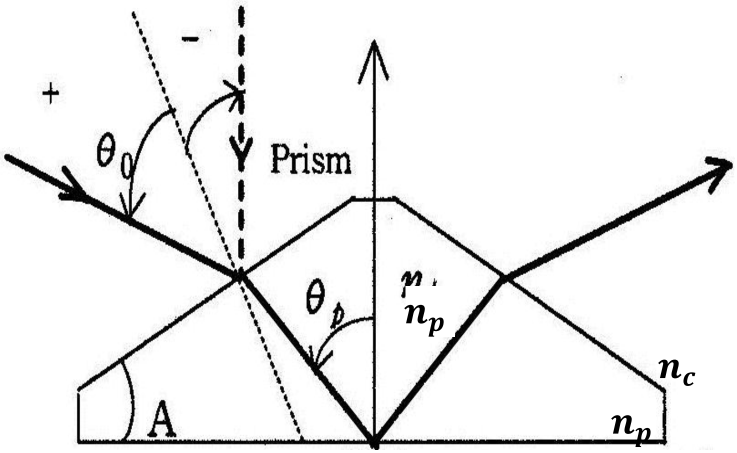
$$r_s = \frac{n_1 cos\theta_i - n_2 cos\theta_t}{n_1 cos\theta_i + n_2 cos\theta_t} = \frac{n_1 cos\theta_i - n_2\sqrt{1-\left(\frac{n_1}{n_2} sin\theta_i\right)^2}}{n_1 cos\theta_i + n_2\sqrt{1-\left(\frac{n_1}{n_2} sin\theta_i\right)^2}}$$

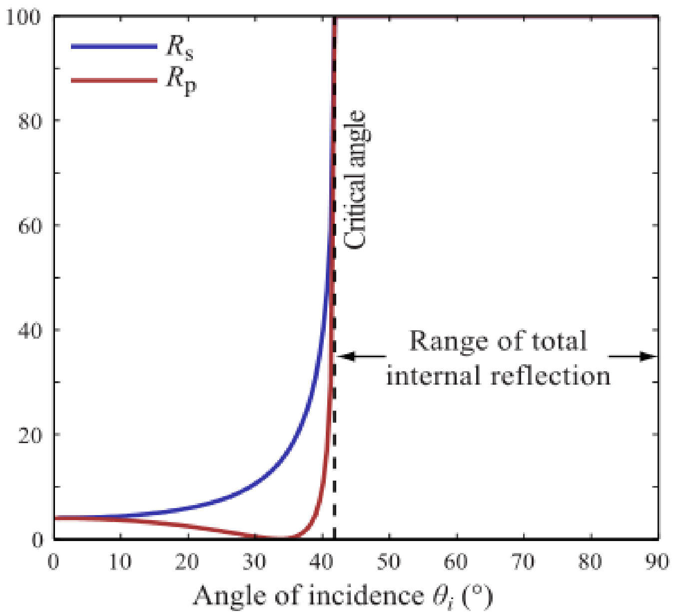
$$r_p = \frac{n_1 cos\theta_t - n_2 cos\theta_i}{n_1 cos\theta_t + n_2 cos\theta_i} = \frac{n_1\sqrt{1-\left(\frac{n_1}{n_2} sin\theta_i\right)^2} - n_2 cos\theta_i}{n_1\sqrt{1-\left(\frac{n_1}{n_2} sin\theta_i\right)^2} + n_2 cos\theta_i}$$

Reflectance

$$R_s = |r_s|^2 \qquad R_p = |r_p|^2$$

Total reflection





Snell's law $n_1 sin\theta_i = n_2 sin\theta_t$

Critical angle $n_1 sin\theta_c = n_2 sin 90°$

$sin\theta_c = \frac{n_2}{n_1}$ $\theta_c = \sin^{-1}\frac{n_2}{n_1}$

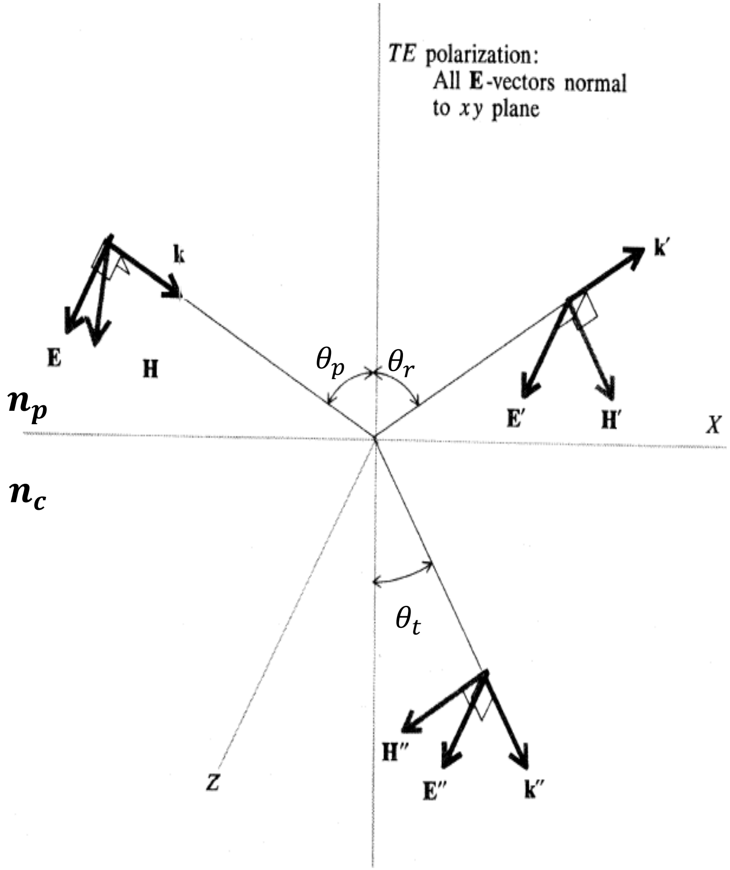
$\boldsymbol{n_p > n_a}$ $\boldsymbol{\theta_p > sin^{-1}\frac{n_a}{n_p}}$

$$r_s = \frac{n_1 cos\theta_i - i\, n_2\sqrt{\left(\frac{n_1}{n_2}sin\theta_i\right)^2 - 1}}{n_1 cos\theta_i + i\, n_2\sqrt{\left(\frac{n_1}{n_2}sin\theta_i\right)^2 - 1}}$$

$$r_p = \frac{i\, n_1\sqrt{\left(\frac{n_1}{n_2}sin\theta_i\right)^2 - 1} - n_2 cos\theta_i}{i\, n_1\sqrt{\left(\frac{n_1}{n_2}sin\theta_i\right)^2 - 1} + n_2 cos\theta_i}$$

Evanescent wave

evanescent waves are formed when waves traveling in a medium undergo total internal reflection at its boundary



$$E_{trans} = E^{"} e^{i(k^{"} \cdot r - \omega t)}$$

$$k^{"} \cdot r = k^{"} x \sin\theta_t - k^{"} z \cos\theta_t$$

$$= k^{"} x \sin\theta_t - i k^{"} z \sqrt{\frac{n_p^2 \sin^2\theta_t}{n_c^2} - 1}$$

$$E_{trans} = E^{"} e^{-\alpha|z|} e^{i(k_1 \cdot r - \omega t)}$$

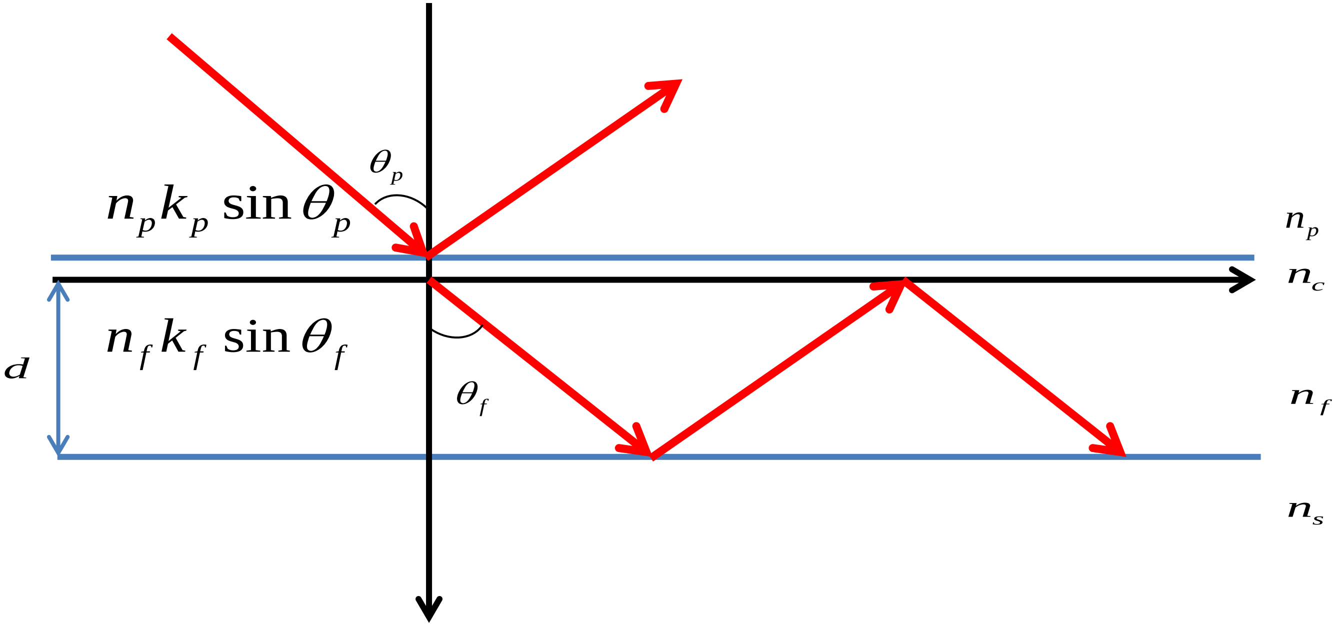
$$\alpha = k^{"} \sqrt{\frac{n_p^2 \sin^2\theta_p}{n_c^2} - 1}$$

$$k_1 = \frac{k^{"} n_p \sin\theta_p}{n_c}$$

Introduction to modern optics 2ed, grant R. Fowles, 43p

Prism coupling

Propagation of electromagnetic wave (in film)



Phase difference = path difference + phase change term = 2mπ (m = 0,1,2,….)

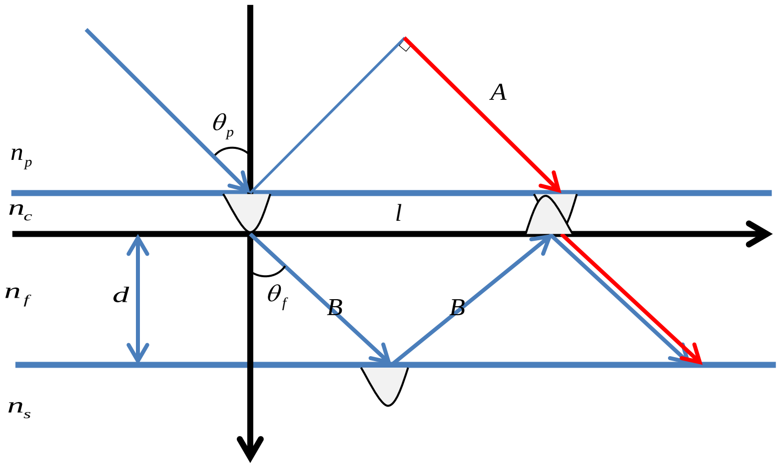
constructive interference = Propagate!

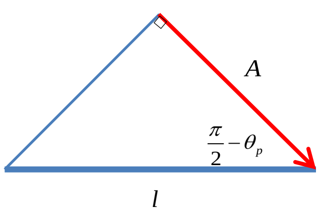
$$2dk_f \cos\theta_f + \varphi_{fs} + \varphi_{fc} = 2m\pi$$ (m = 0, 1, 2, 3, ….)

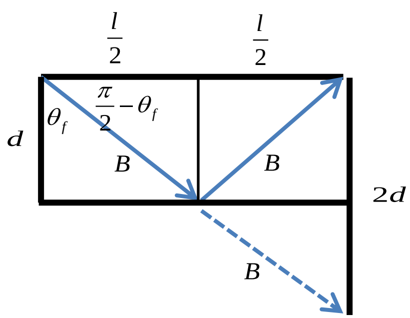
φ_{fs} Film-substrate phase change φ_{fc} Film-airgap phase change

Prism coupling

Calculate phase difference







$$\frac{A}{l} = \sin\theta_p$$

$$A = l\sin\theta_p$$

$$\frac{2d}{l} = \tan\left(\frac{\pi}{2} - \theta_f\right)$$

$$l = 2d\tan\theta_f = 2d\,\frac{\sin\theta_f}{\cos\theta_f}$$

$$2B = \frac{l}{\sin\theta_f}$$

$$\Delta\phi = (k_f\,2B + \varphi_{fs} + \varphi_{fc}) - (k_p A) = 2m\pi$$

φ_{fs} **Film-substrate phase change**

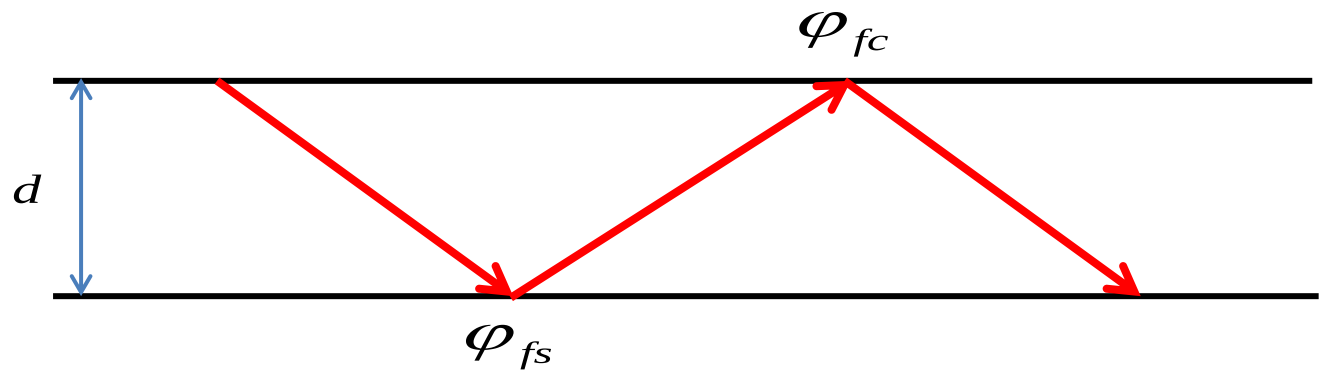
φ_{fc} **Film-airgap phase change**

$$n_p\sin\theta_p = n_f\sin\theta_f$$

$$\Delta\varphi = 2dk_f\cos\theta_f + \varphi_{fs} + \varphi_{fc} = 2m\pi$$

Prism coupling

phase change



S-wave

$$r_s = e^{-i\delta_s} = \frac{ae^{-i\alpha}}{ae^{+i\alpha}}$$

$$ae^{i\alpha} = \cos\theta + i\sqrt{\sin^2\theta - (\frac{n_2}{n_1})^2}$$

$$r_s = e^{-i\delta_s} = e^{-i2\alpha}$$

$$\delta_s = 2\alpha$$

$$\tan\alpha = \tan\frac{\delta_s}{2} = \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}$$

Prism coupling

$$\frac{4\pi d}{\lambda} n_f \cos\theta_f + \varphi_{fs} + \varphi_{fc} = 2m\pi$$

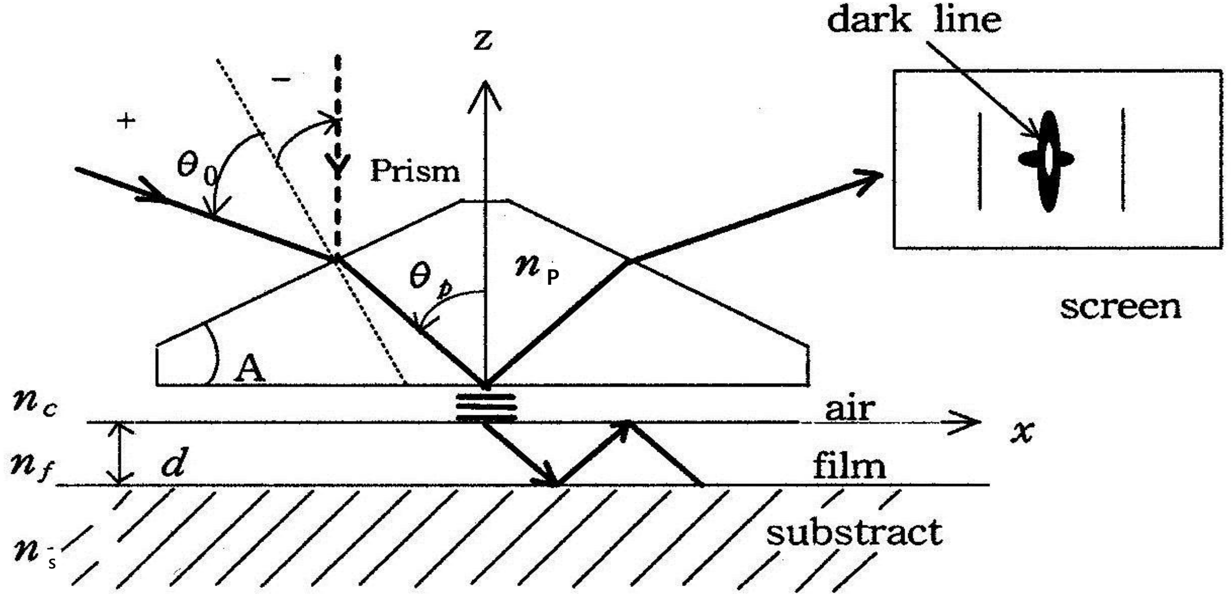
$$N_{eff} = n_f \sin\theta_f$$

$$\frac{4\pi d}{\lambda}\sqrt{n_f^2 - N_{eff}^2} - 2\arctan\left(f_s \sqrt{\frac{N_{eff}^2 - n_s^2}{n_f^2 - N_{eff}^2}} \right) - 2\arctan\left(f_c \sqrt{\frac{N_{eff}^2 - n_c^2}{n_f^2 - N_{eff}^2}} \right) = 2m\pi$$

$$f_s = \begin{cases} 1: & s - wave \\ \dfrac{n_f^2}{n_s^2}: & p - wave \end{cases} \quad , \quad f_c = \begin{cases} 1: & s - wave \\ \dfrac{n_f^2}{n_c^2}: & p - wave \end{cases}$$

Prism coupling

Effective refractive index



$$(\theta_{p'} + \frac{\pi}{2}) + A + (\frac{\pi}{2} - \theta_p) = \pi \longrightarrow \theta_p = \theta_{p'} + A$$

Prism coupling

Effective refractive index

$$N_{eff} = n_f \sin\theta_f$$

$$N_{eff} = n_f \sin\theta_f$$

$$= n_p \sin\theta_p$$

$$\theta_p = \theta_{p'} + A$$

$$= n_p \sin(\theta_p' + A)$$

$$= n_p \sin\theta_p' \cos A + n_p \cos\theta_p' \sin A$$

$$n_p \sin\theta_p = \sin\theta_0$$

$$N_{eff} = \sin A\sqrt{n_p^2 - \sin\theta_0^2} + \cos A \sin\theta_0$$

Prism coupling

Calculate n_f , d

$$\frac{4\pi d}{\lambda}\sqrt{n_f^2(m)-N_{eff}^2(m)}=F(m)$$

$$F(m)=\arctan\left(f_s\sqrt{\frac{N_{eff}^2-n_s^2}{n_f^2-N_{eff}^2}}\right)+\arctan\left(f_c\sqrt{\frac{N_{eff}^2-n_c^2}{n_f^2-N_{eff}^2}}\right)+m\pi$$

$$n_f=\sqrt{\frac{N_{eff}^2(p)F^2(q)-N_{eff}^2(q)F^2(p)}{F^2(q)-F^2(p)}}$$

(m = p , q)

$$d=\frac{\lambda F(p)}{2\pi\sqrt{n_f^2-N_{eff}^2(p)}}$$