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A Newtonian approach to extraordinarily strong negative refraction

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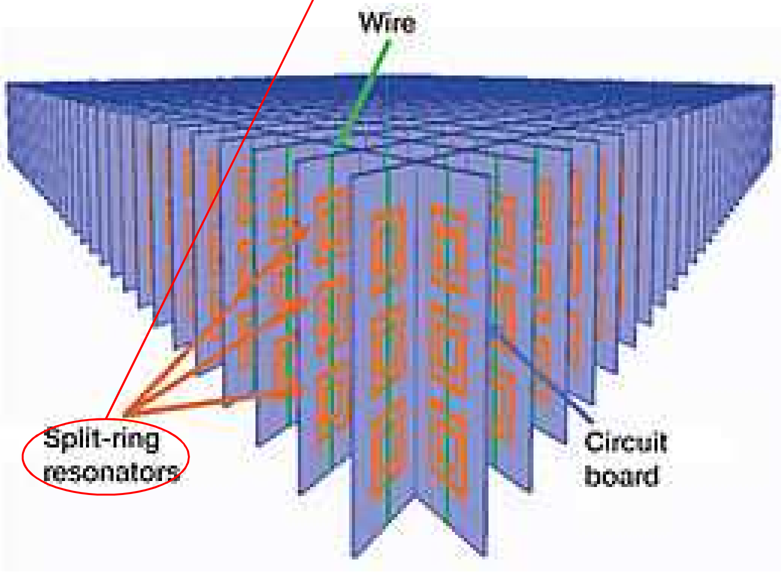
Metamaterial and negative refraction

Metamaterial – having both $\mu < 0, \varepsilon < 0$

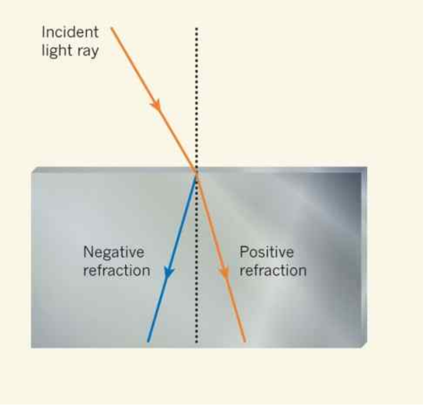
$$n = \sqrt{\mu_r \varepsilon_r} < 0$$

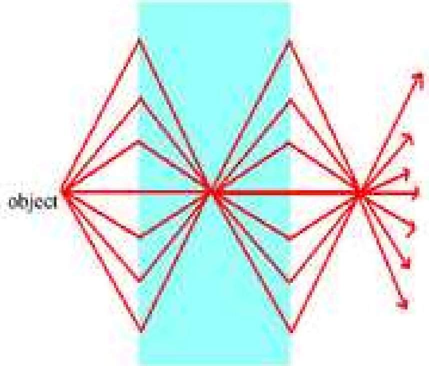


http://en.wikipedia.org/wiki/Split-ring_resonator

<Super lens>

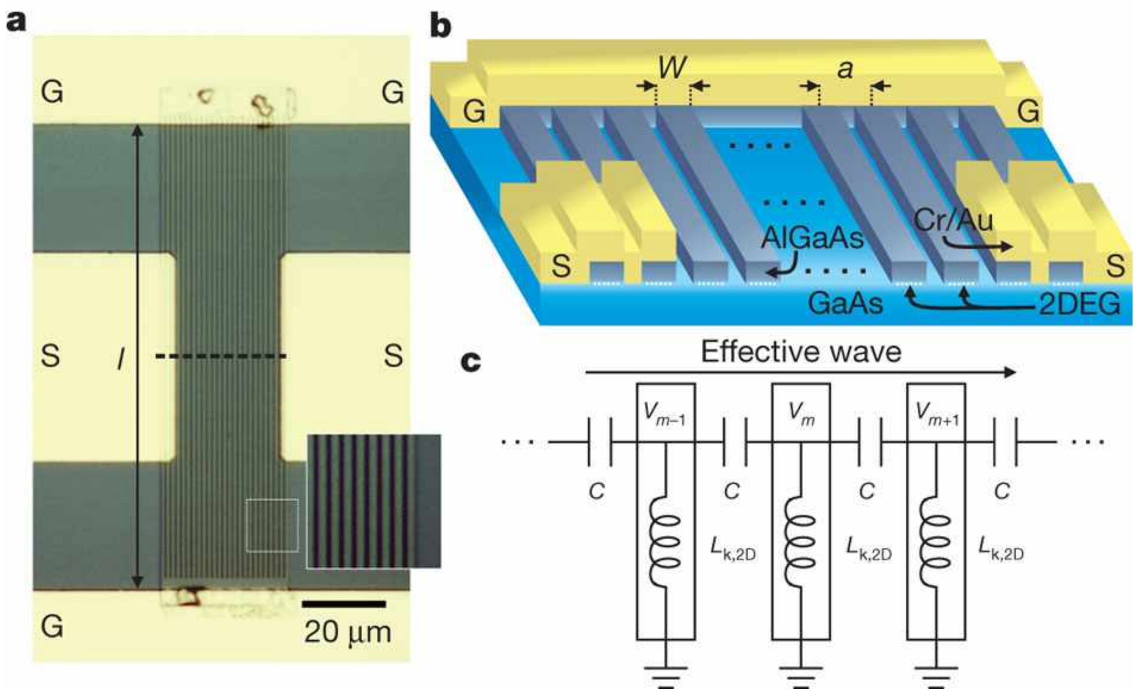
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$



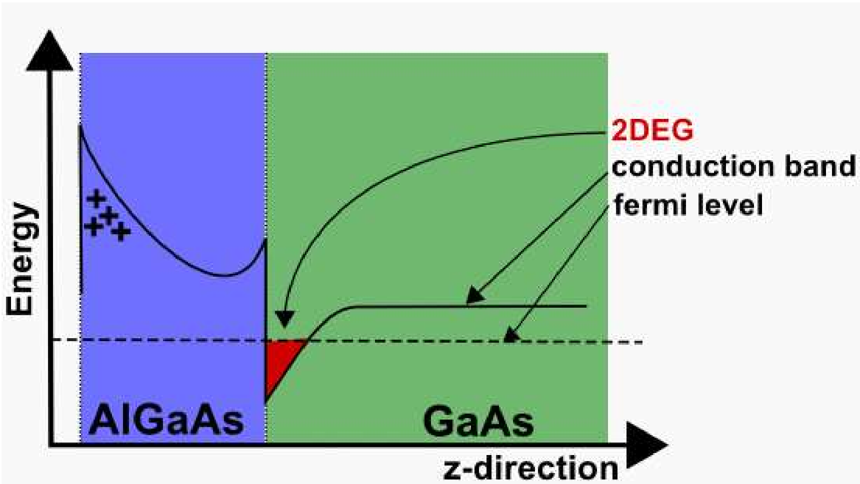


http://www.nature.com/nature/journal/v480/n7375/fig_tab/480042a_F1.html

2D-electron system in semiconductor strip

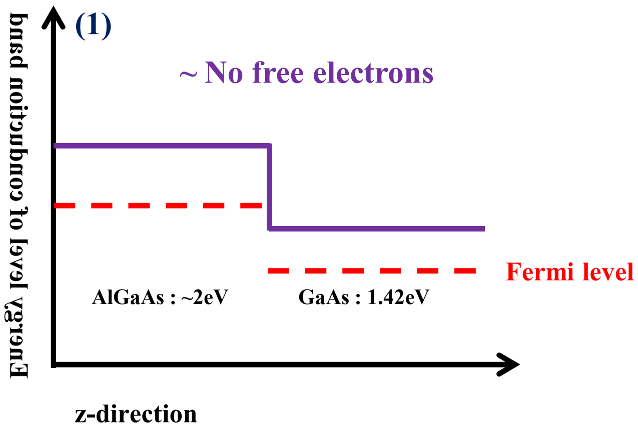


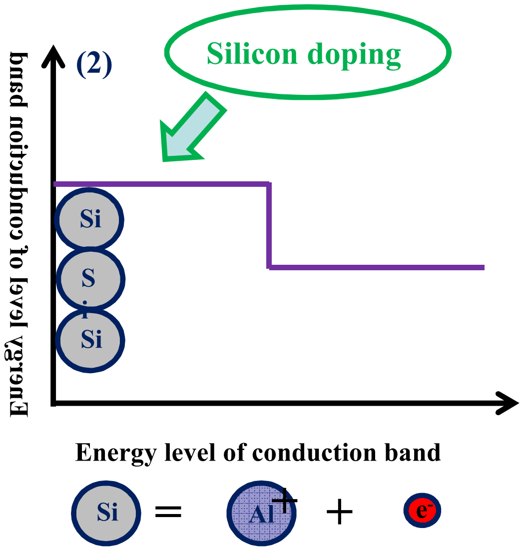
l=112μm, a=1.25μm, W=1μm and number of strip ~13

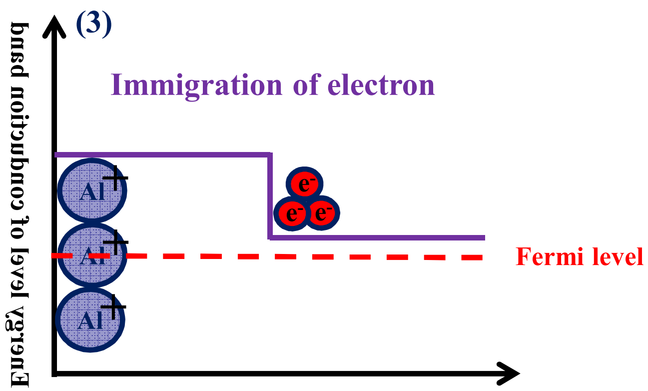


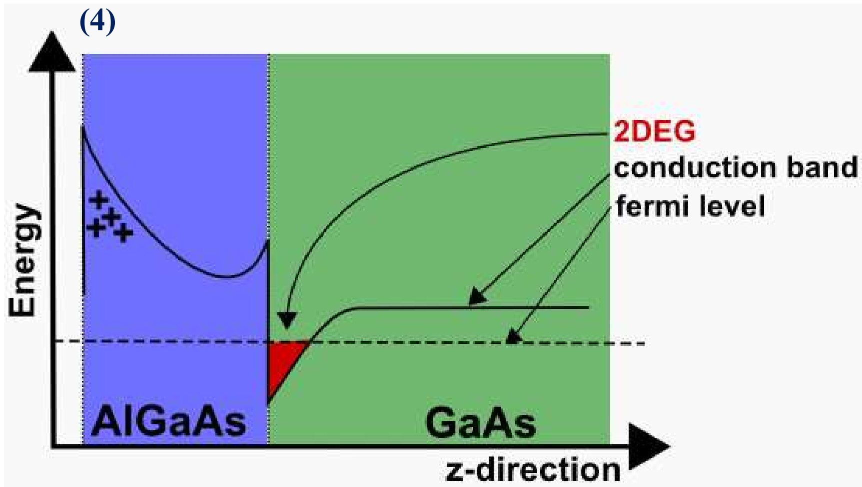
Motion of 2DEG is induced by external EM field. (along xy-plane)

Band bending and modulation dopping









2D-electron motion and kinetic inductance

$$-e\frac{V}{l}-\frac{m^*v}{\tau}=m^*\frac{dv}{dt}$$: Equation of motion

Under AC field (eg. Electromagnetic wave) $V(x,t)=Ve^{i(kx-wt)}$

$$-e\frac{V}{l}-\frac{m^*v}{\tau}=m^*i\omega v$$

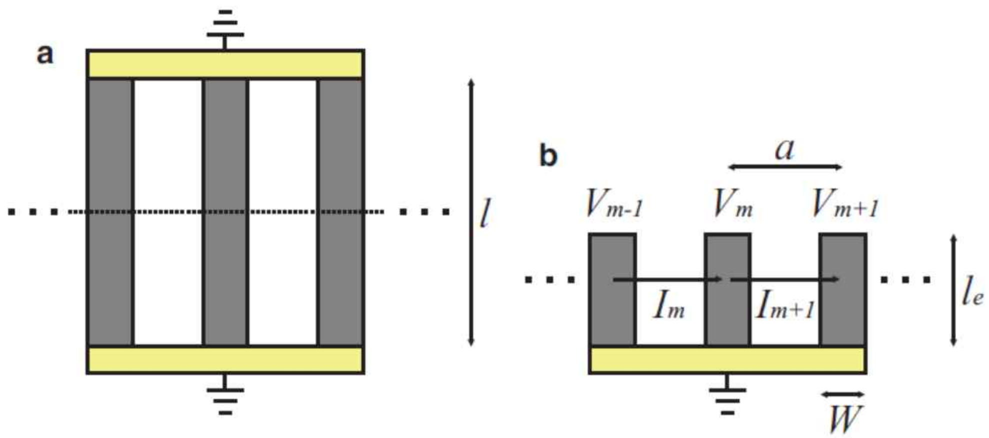
With a definition of current in this system $I=-n_{2D}evW$

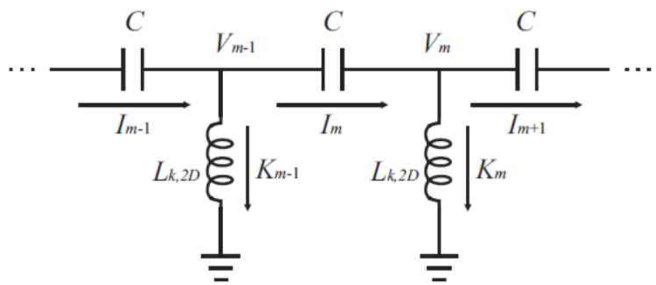
$$\frac{V}{I}=i\omega\frac{m^*}{n_{2D}e^2}\times\frac{l}{W}+\frac{m^*}{n_{2D}e^2\tau}\times\frac{l}{W}.$$: expression of impedance

$$Z=iwL_{k,2D}+R$$

$$\boxed{L_{k,2D}=\frac{m^*}{n_{2D}e^2}\times\frac{l}{W}. \qquad R_{2D}=\frac{m^*}{n_{2D}e^2\tau}\times\frac{l}{W}=\frac{1}{n_{2D}e\mu}\times\frac{l}{W}}$$

Derivation of refractive index from $L_{2D,k}$





With circuit equations in this system,

$$C\frac{\mathrm{d}}{\mathrm{d}t}(V_{m-1}-V_m)=I_m \qquad I_m-I_{m+1}=K_m \qquad V_m=L_{k,2D}\frac{dK_m}{dt}$$

$$\boxed{\frac{1}{L_{k,2D}C}V_m=\frac{d^2}{dt^2}(V_{m-1}+V_{m+1}-2V_m)}$$

After inserting V_{in} induced by electromagnetic wave, $V_m = V_0 e^{i(\omega t - kma)}$

Derivation of refractive index from $L_{2D,k}$

$$-w^2 V_0 e^{(wt-kma)}(e^{ika}+e^{-ika}-2)=\frac{1}{L_{K,2D}C}V_0 e^{(wt-kma)}$$

So, dispersion relation can be expressed as,

$$\omega(k)=\frac{1}{2\sqrt{L_{k,2D}C}}\left|\sin\frac{ka}{2}\right|^{-1}=\omega_c\left|\sin\frac{ka}{2}\right|^{-1},$$

$$\omega_c \equiv \frac{1}{2\sqrt{L_{k,2D}C}}$$

From definition of refractive index, $n=\frac{\mathrm{sgn}(d\omega/dk)}{\mathrm{sgn}(\omega/k)}\times|k|c/\omega,$

$$\boxed{\underline{n}=-\frac{2c}{a\omega}\sin^{-1}\left(\frac{\omega_c}{\omega}\right)}$$

Negative value if $w>w_c$

Comparison between 2D and 3D system

Kinetic and magnetic inductance of single 3D gold nano particle

A	$L_{k,3D}/l$	L_m/l	$L_{k,3D}/L_m$
1×1 nm^2	600 pH/μm	$\sim$ 1.4 pH/μm	430
5×5 nm^2	24 pH/μm	$\sim$ 1.2 pH/μm	20
10×10 nm^2	6.0 pH/μm	$\sim$ 1.0 pH/μm	6
20×20 nm^2	1.5 pH/μm	$\sim$ 0.9 pH/μm	1.7

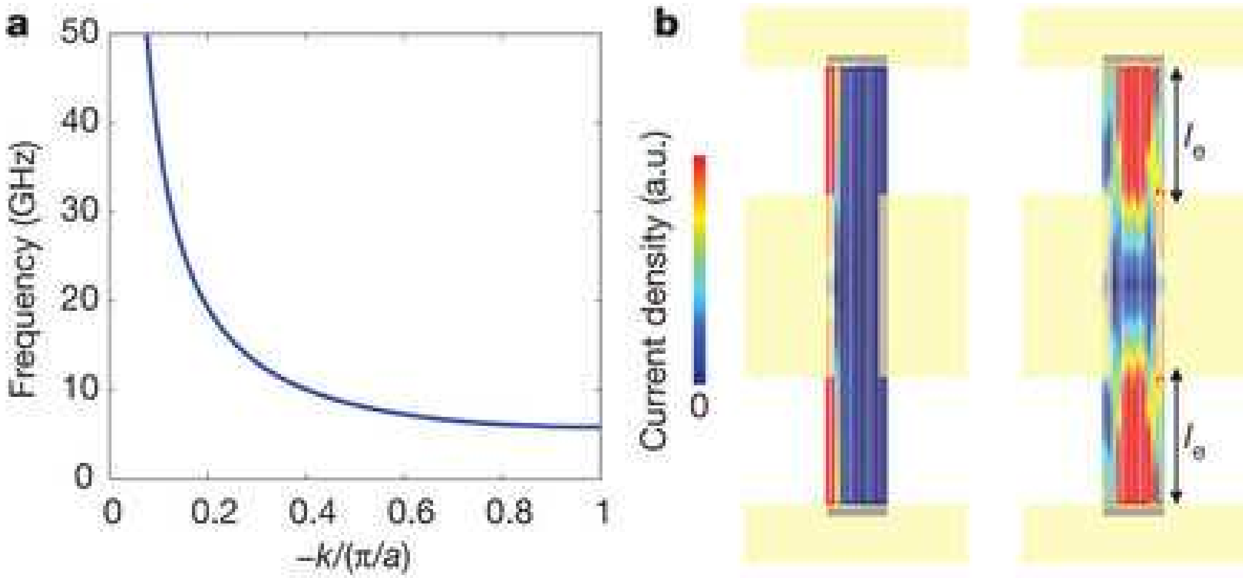
Kinetic and magnetic inductance of 2DEG strip array

W	$L_{k,2D}/l$	L_m/l	$L_{k,2D}/L_m$
20 nm	62.5 nH/μm	$\sim$ 2.3 pH/μm	27,000
100 nm	12.5 nH/μm	$\sim$ 2.0 pH/μm	6,300
500 nm	2.50 nH/μm	$\sim$ 1.6 pH/μm	1,600
1000 nm	1.25 nH/μm	$\sim$ 1.4 pH/μm	890

→ 2DEG strip system has larger kinetic inductance

* Carrier density for silver is around 1×10^{28} m^{-3}

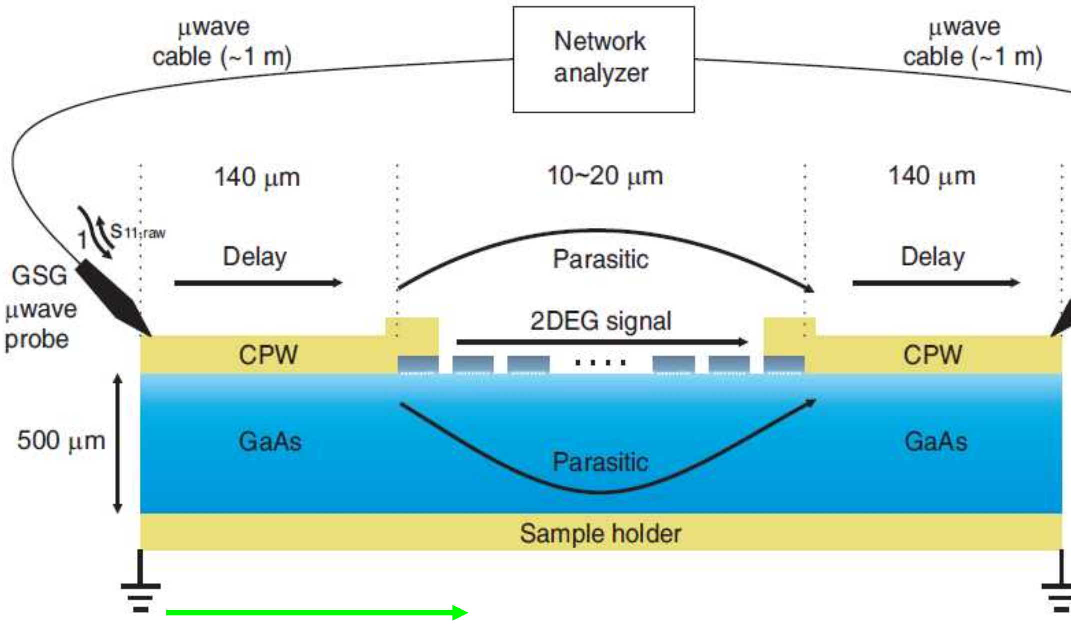
Result of simulated dispersion curve and current

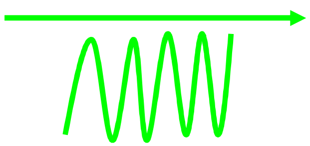


Dispersion curve indicates that group and phase velocity have opposite sign.

Frequency above cutoff creates high current density on the stripes.

Measurement of microwave



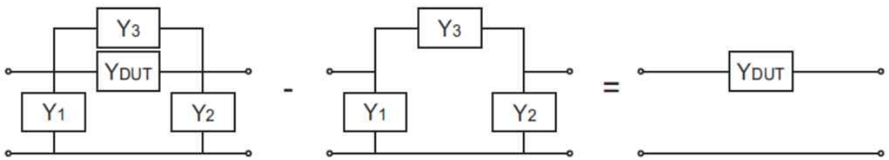


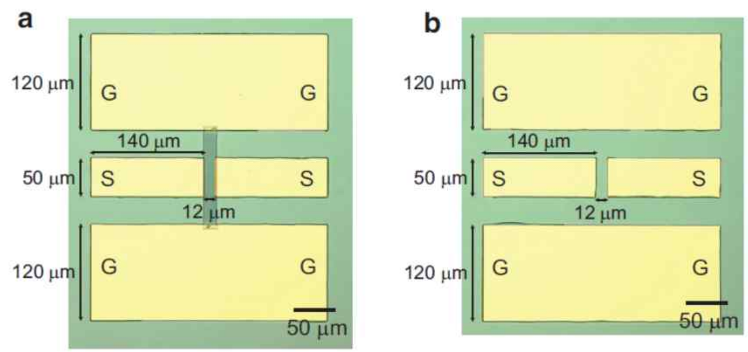
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\boxed{\begin{aligned} e^{ikd} &= X \pm i\sqrt{1 - X^2}, \\ X &= \frac{1 - s_{11}^2 + s_{21}^2}{2 s_{21}}. \end{aligned}}$$

Effective wavenumber can be extracted from S parameters

$$n = \frac{\mathrm{sgn}(d\omega/dk)}{\mathrm{sgn}(\omega/k)} \times |k| c/\omega,$$





Transfer and scattering matrix

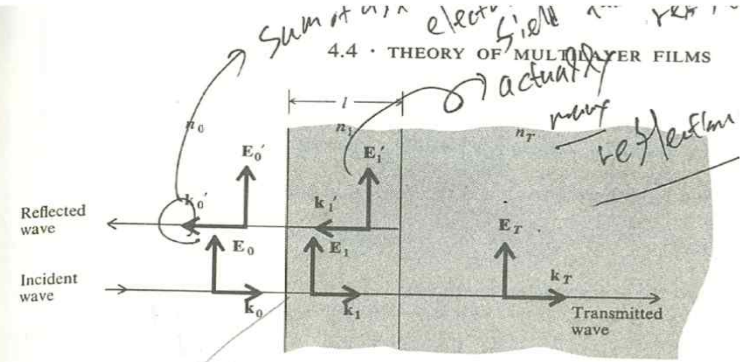


Figure 4.7. Wave-vectors and their associated electric fields for the case of normal incidence on a single dielectric layer.

cidence are easily made. The amplitude of the electric vector of the incident beam is E_0. That of the reflected beam is E_0', and that of the transmitted beam is E_T. The electric-field amplitudes in the film are E_1 and E_1' for the forward and backward traveling waves, respectively, as indicated in the figure.

The boundary conditions require that the electric and magnetic fields be continuous at each interface. These conditions are expressed as follows:

First Interface	*Second Interface*
Electric: $E_0 + E_0' = E_1 + E_1'$	$E_1 e^{ikl} + E_1' e^{-ikl} = E_T$
Magnetic: $H_0 - H_0' = H_1 - H_1'$	$H_1 e^{ikl} - H_1' e^{-ikl} = H_T$
or $n_0 E_0 - n_0 E_0' = n_1 E_1 - n_1 E_1'$	$n_1 E_1 e^{ikl} - n_1 E_1' e^{-ikl} = n_T E_T$

The relations for the magnetic fields follow from the theory developed in Section 2.7. The phase factors e^{ikl} and e^{-ikl} result from the fact that the wave travels through a distance l from one interface to the other.

If we eliminate the amplitudes E_1 and E_1', we obtain

$$1 + \frac{E_0'}{E_0} = \left(\cos kl - i\frac{n_T}{n_1}\sin kl\right)\frac{E_T}{E_0}$$
$$n_0 - n_0\frac{E_0'}{E_0} = (-in_1 \sin kl + n_T \cos kl)\frac{E_T}{E_0} \tag{4.22}$$

or, in matrix form,

$$\begin{bmatrix}1\\n_0\end{bmatrix} + \begin{bmatrix}1\\-n_0\end{bmatrix}\frac{E'_0}{E_0} = \begin{bmatrix}\cos kl & \frac{-i}{n_1}\sin kl\\ -in_1 \sin kl & \cos kl\end{bmatrix}\begin{bmatrix}1\\n_T\end{bmatrix}\frac{E_T}{E_0} \tag{4.23}$$

which can be abbreviated as

$$\begin{bmatrix}1\\n_0\end{bmatrix} + \begin{bmatrix}1\\-n_0\end{bmatrix} r = M\begin{bmatrix}1\\n_T\end{bmatrix} t \tag{4.24}$$

We have here introduced the reflection coefficient

$$r = \frac{E_0'}{E_0} \tag{4.25}$$

and the transmission coefficient

$$t = \frac{E_T}{E_0} \tag{4.26}$$

The matrix, known as the *transfer matrix*

$$M = \begin{bmatrix}\cos kl & -\frac{i}{n_1}\sin kl\\ -in_1 \sin kl & \cos kl\end{bmatrix} \tag{4.27}$$

where n_1 is the index of refraction, and $k = 2\pi/\lambda = 2\pi n_1/\lambda_0$.

Now suppose that we have N layers numbered 1, 2, 3, . . . N having indices of refraction $n_1, n_2, n_3, \ldots n_N$ and thicknesses $l_1, l_2, l_3, \ldots l_N$, respectively. In the same way that we derived Equation (4.24), we can show that the reflection and transmission coefficients of the multilayer film are related by a similar matrix equation:

$$\begin{bmatrix}1\\n_0\end{bmatrix} + \begin{bmatrix}1\\-n_0\end{bmatrix} r = M_1 M_2 M_3 \cdots M_N \begin{bmatrix}1\\n_T\end{bmatrix} t = M\begin{bmatrix}1\\n_T\end{bmatrix} t \tag{4.28}$$

where the transfer matrices of the various layers are denoted by M_1, $M_2, M_3, \ldots, M_N$. Each transfer matrix is of the form given by Equation (4.27) with appropriate values of n, l, and k. The overall transfer matrix M is the product of the individual transfer matrices. Let the elements of M be A, B, C, and D, that is

$$M_1 M_2 M_3 \cdots M_N = M = \begin{bmatrix}A & B\\C & D\end{bmatrix} \tag{4.29}$$

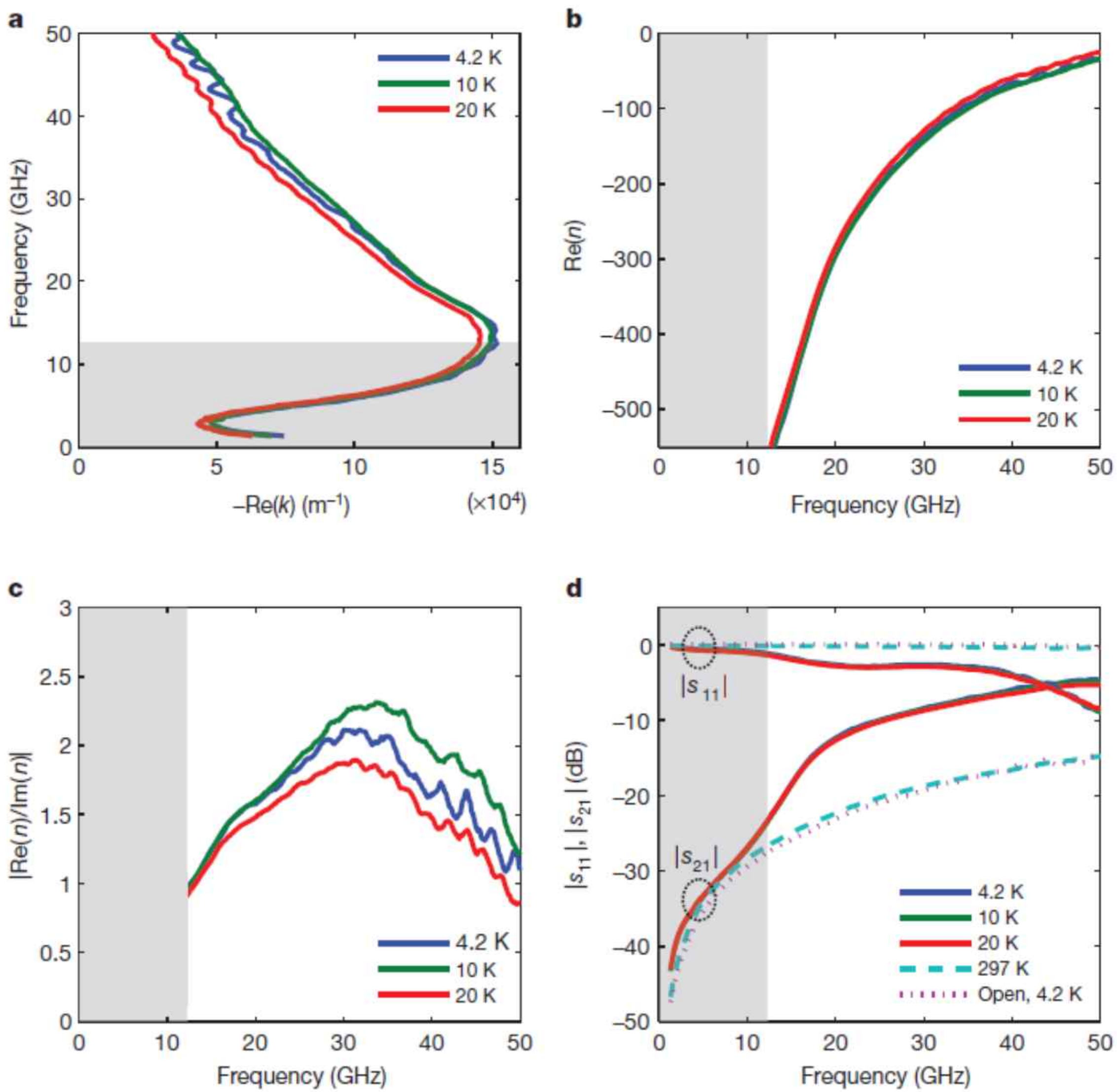
We can then solve Equation (4.28) for r and t in terms of these elements. The result is

$$r = \frac{An_0 + Bn_T n_0 - C - Dn_T}{An_0 + Bn_T n_0 + C + Dn_T} \tag{4.30}$$

$$t = \frac{2n_0}{An_0 + Bn_T n_0 + C + Dn_T} \tag{4.31}$$

The reflectance R and the transmittance T are then given by $R = |r|^2$ and $T = |t|^2$, respectively.

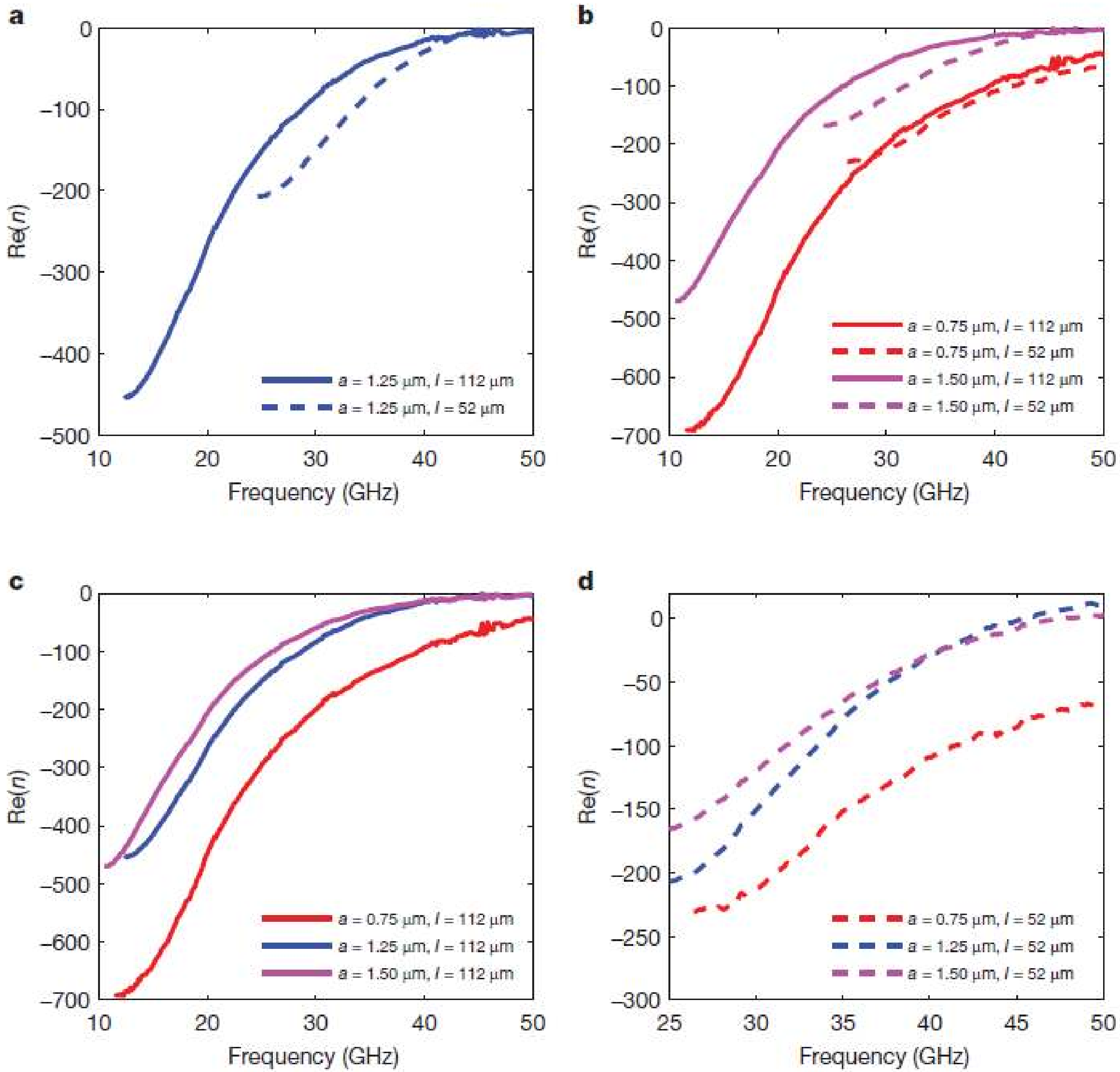
Temperature dependence



In room temperature, strip has high resistance (~100kOhm)

Kinetic inductance is reduced and system acts as open circuit

Effect of system dimension



$$L_{k,2D} = \frac{m^*}{n_{2D}e^2} \times \frac{l}{W}.$$

$$n = -\frac{2c}{a\omega}\sin^{-1}\left(\frac{\omega_c}{\omega}\right)$$

Summary

1. From the metamaterial having 2DEG Strong negative refractive index was observed in GHz range.

2. Origin of negative refraction dominantly came from collective motion of electrons in the strip array which related to kinetic inductance.

3. Temperature and size dependence of n was studied.