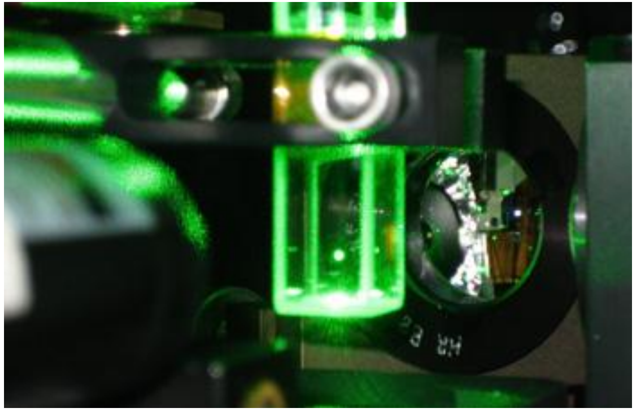
Was Einstein Wrong? Part II

- Measurement of the Instantaneous Velocity of a Brownian Particle



Seoncheol Cha
Soft Matter Optical Spectroscopy, Department of Physics, Sogang Univeristy

Robert Brown found the random motion (1827)

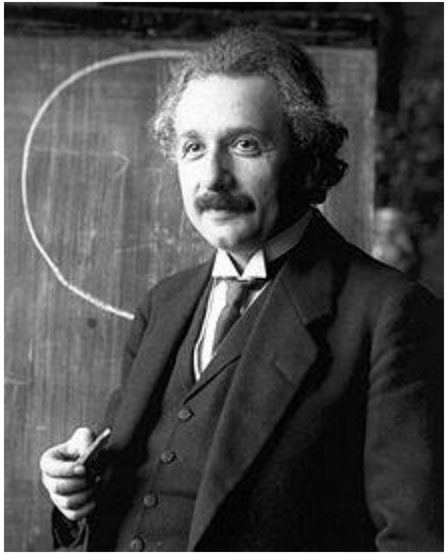
The origin of random motion was explained as the amplification of the statistical fluctuations of the surrounding fluid molecules

The length of the path travelled in a given time is unknown

The mean square displacement (MSD) can be measured
$MSD(t) = 2Dt$

D : particle's diffusion coefficient

Albert Einstein explains the random motion (1905)



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Langevin equation (1908)

The interaction of the particle with the surrounding fluid
must break down at short time scale

-> particle's inertia becomes significant
"ballistic motion"

Characteristic time scale $\tau_p = m/\gamma$
m : mass of spherical particle
γ : Stokes viscous drag coefficient

$MSD(t) = (k_B T/m)t^2$

Hard-sphere simulations & Experiments (1960s)

Even for timescales much larger $\tau_p = m/\gamma$, Einstein's description already fails.

"Hydrodynamic memory effect" : originate from the inertia of the surrounding fluid, which leads to long-lived vortices caused by and in turn affecting the particle's motion

Introduce an intermediate regime between the purely ballistic t^2 and the diffusive 2Dt scaling

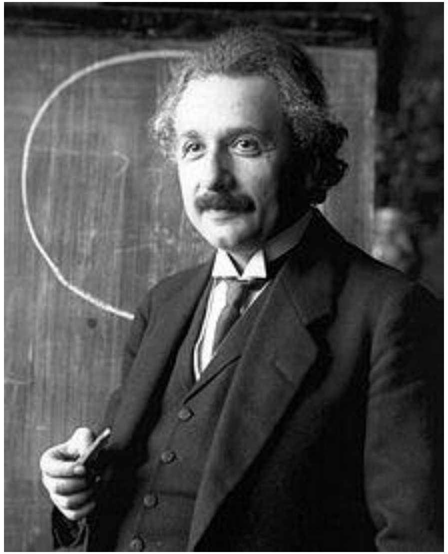
$$\tau_f = r^2\rho_f/\eta$$

r : particle's radius
ρ_f : density of the fluids
η : viscosity of the fluids

$$\tau_f / \tau_p = 9\rho_f / 2\rho_p$$

ρ_p: particles' density

Experimental challenges



Einstein declared that, due to the rapid deceleration caused by the viscosity of the medium, it would be impossible to measure the instantaneous velocity of an ultramicroscopic Brownian particle moving in a liquid.

Measurement in a gas phase

Viscosity is more than 50 times lower

The timescale of inertial movement are increased

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Measurement of the Translational and Rotational Brownian Motion of Individual Particles in a Rarefied Gas

Jürgen Blum,* Stefan Bruns, Daniel Rademacher, Annika Voss, Björn Willenberg, and Maya Krause

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(Received 2 August 2006; published 4 December 2006)

The transition from diffusive to ballistic motion was measured by video microscopy

Measurement in a gas phase

Sciencexpress Report

Measurement of the Instantaneous Velocity of a Brownian Particle

Tongcang Li, Simon Kheifets, David Medellin, Mark G. Raizen*
Center for Nonlinear Dynamics and Department of Physics, University of Texas at Austin, Austin, TX 78712, USA.

Brownian particle's instantaneous velocities follow the Maxwell distribution

Measurement in a gas phase

The solution to the Langevin equation was fully sufficient to describe the experimental results in a gas phase

Liquid Phase ?

ARTICLES
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nature physics

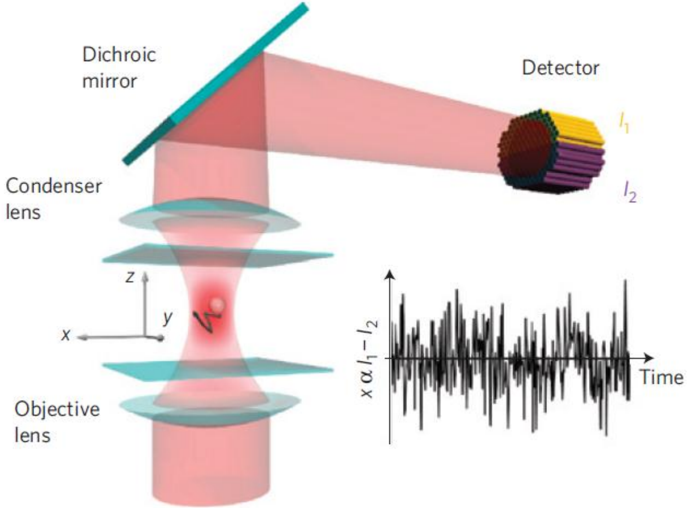
Direct observation of the full transition from ballistic to diffusive Brownian motion in a liquid

Rongxin Huang[1], Isaac Chavez[1], Katja M. Taute[1], Branimir Lukić[2], Sylvia Jeney[2], Mark G. Raizen[1] and Ernst-Ludwig Florin[1]*

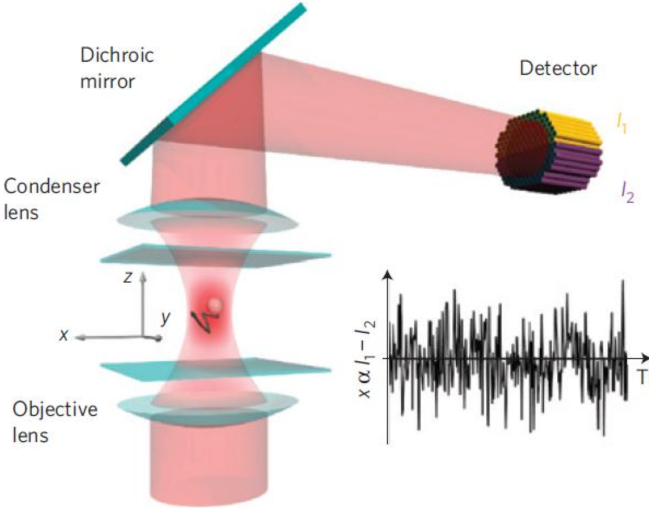
At timescales once deemed immeasurably small by Einstein, the random movement of Brownian particles in a liquid is expected to be replaced by ballistic motion. So far, an experimental verification of this prediction has been out of reach due to a lack of instrumentation fast and precise enough to capture this motion. Here we report the observation of the Brownian motion of a single particle in an optical trap with 75 MHz bandwidth and sub-ångström spatial precision and the determination of the particle's velocity autocorrelation function. Our observation is the first measurement of ballistic Brownian motion of a particle in a liquid. The data are in excellent agreement with theoretical predictions taking into account the inertia of the particle and hydrodynamic memory effects.

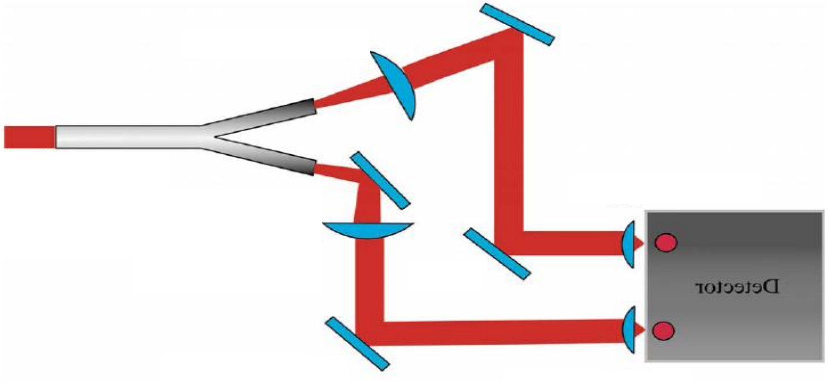
Setup

Glass, polystyrene, and melamine resin beads
1 particle / 200 x 200 x 200 μm^3
Trapping Beam : 1064 nm, 600 mW

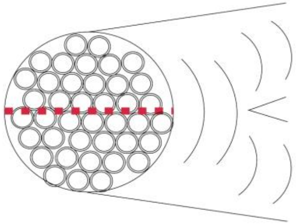


Setup





Sampling rate
75MHz



1000 multimode fibers
105 μm core, 125 μm cladding

-> 4.4 mm diameter fiber splitter

Raizen, Rev. Sci. Instrum (2008)

Setup

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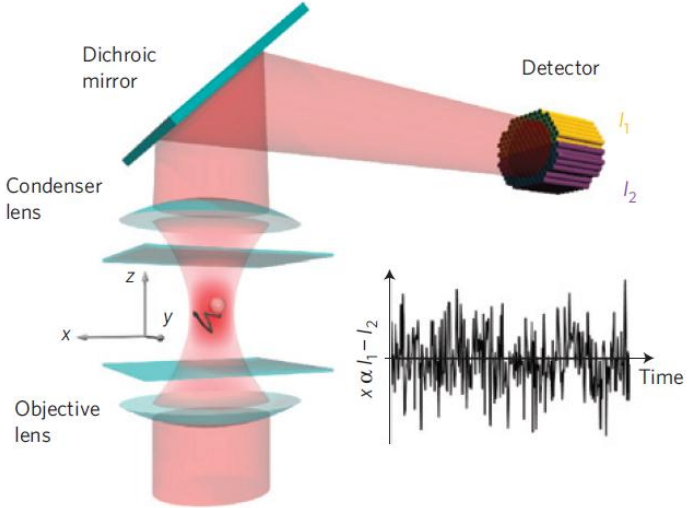
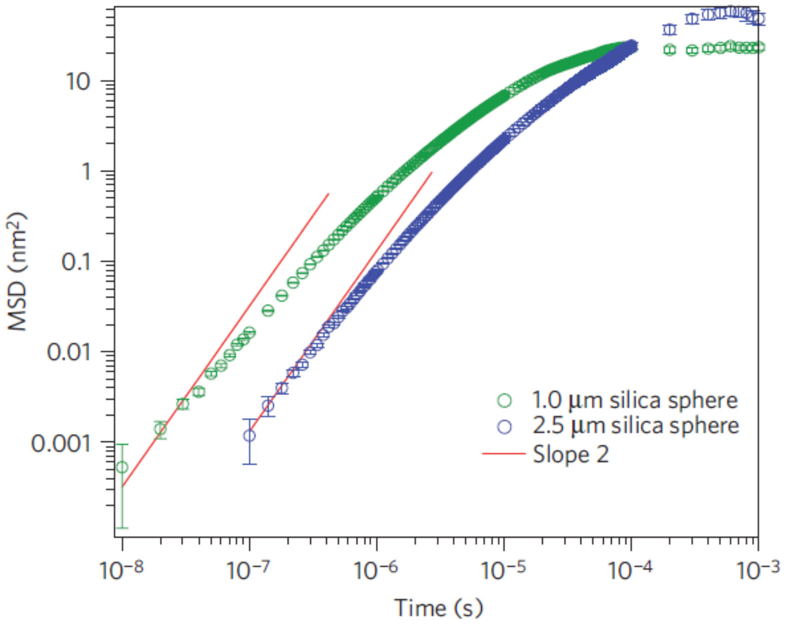
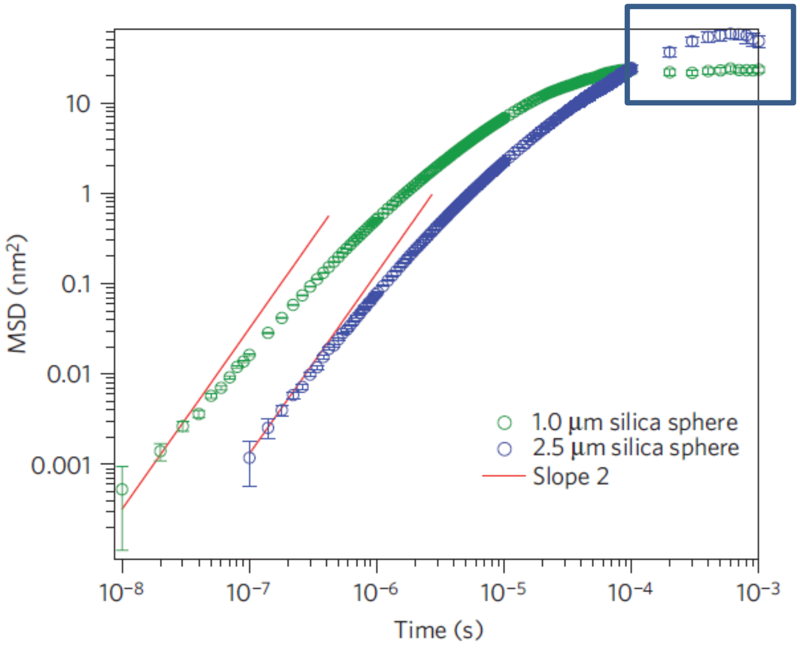


MSD - Performances



MSD is measured from 10 ns to 1 ms
Temporal resolution ~ 10 ns
Resolve a MSD as small as ~0.0005 nm^2
(20 pm spaital resolution ~ size of H atom)

MSD - Optical trap effect

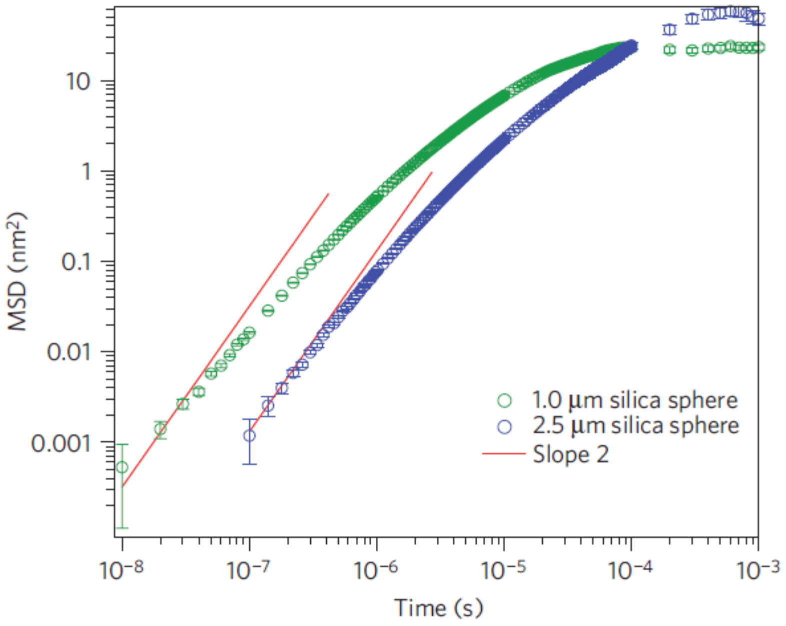


Due to the optical trap

$\tau_K = \gamma / K$

K : the stiffness of the optical trapping ptl.

MSD – ballistic motion

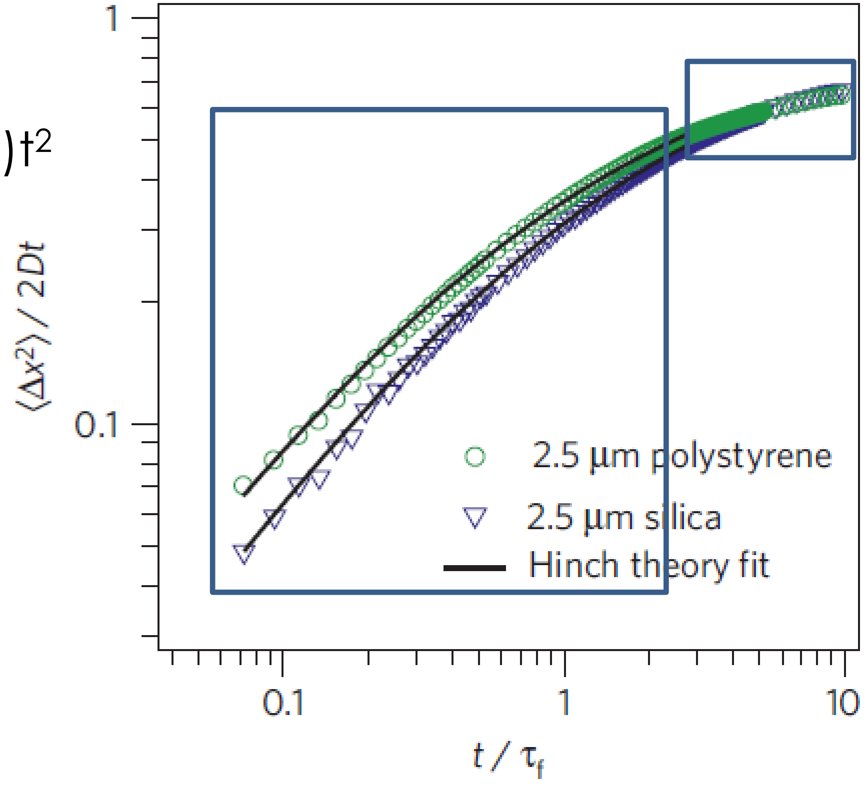


$\tau_f = r^2\rho_f/\eta$

Larger bead displays ballistic motion at larger time scale

Evidence for the ballistic motion

$MSD(t) = (k_B T/\mathbf{m})t^2$

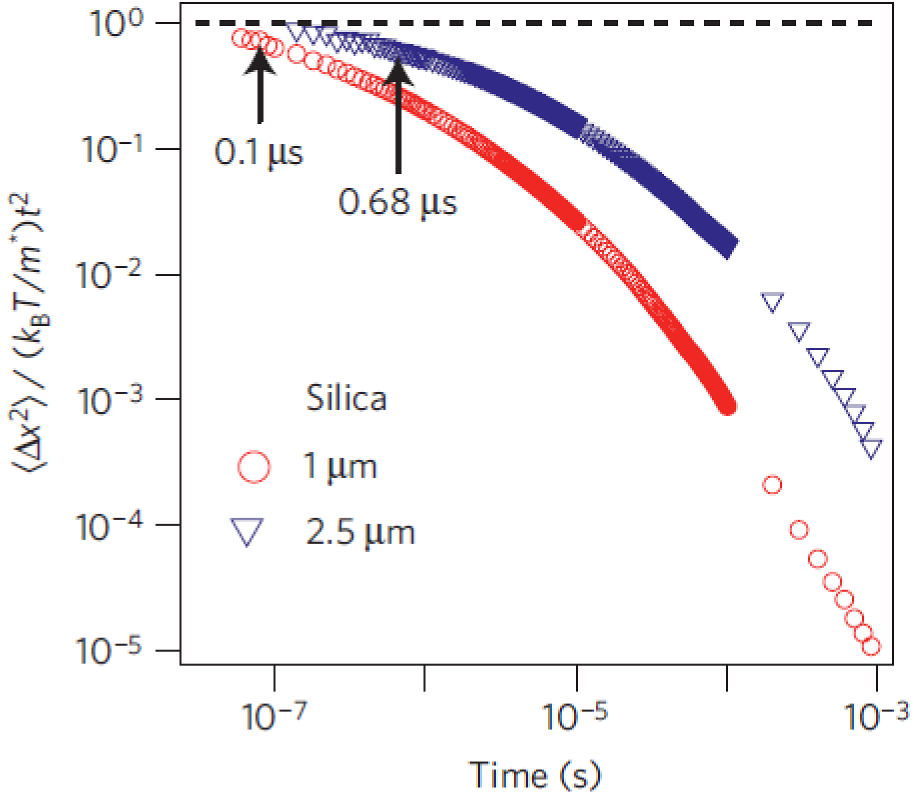


$sMSD(t) = 2Dt$

$\tau_f = r^2 \rho_f / \eta = 1.6 \mu s$

Evidence for the ballistic motion

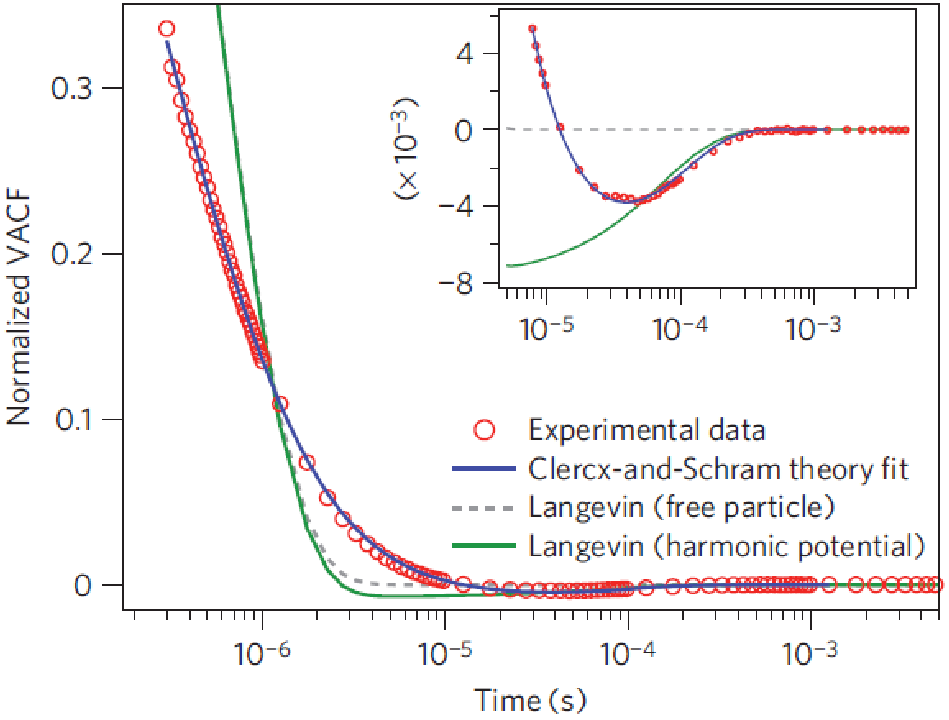
$MSD(t) = (k_B T/\mathbf{m})t^2$



Velocity Autocorrelation Function

elocity autocorrelation function

$Z(t) = \langle u_x(t) u_x \rangle$



Particle-liquid interactions at short time scale

Equaipartition theorm

$m\langle v \rangle^2 = k_B T$

$m^*\langle v \rangle^2 = k_B T$ for short time scale

The effects of the non-negligible compressibility of the fluid at short time scales.

-> assuming m* effective mass, fluid to be incompressible

Time scale ~ 0.3 ns / displacement 1 pm