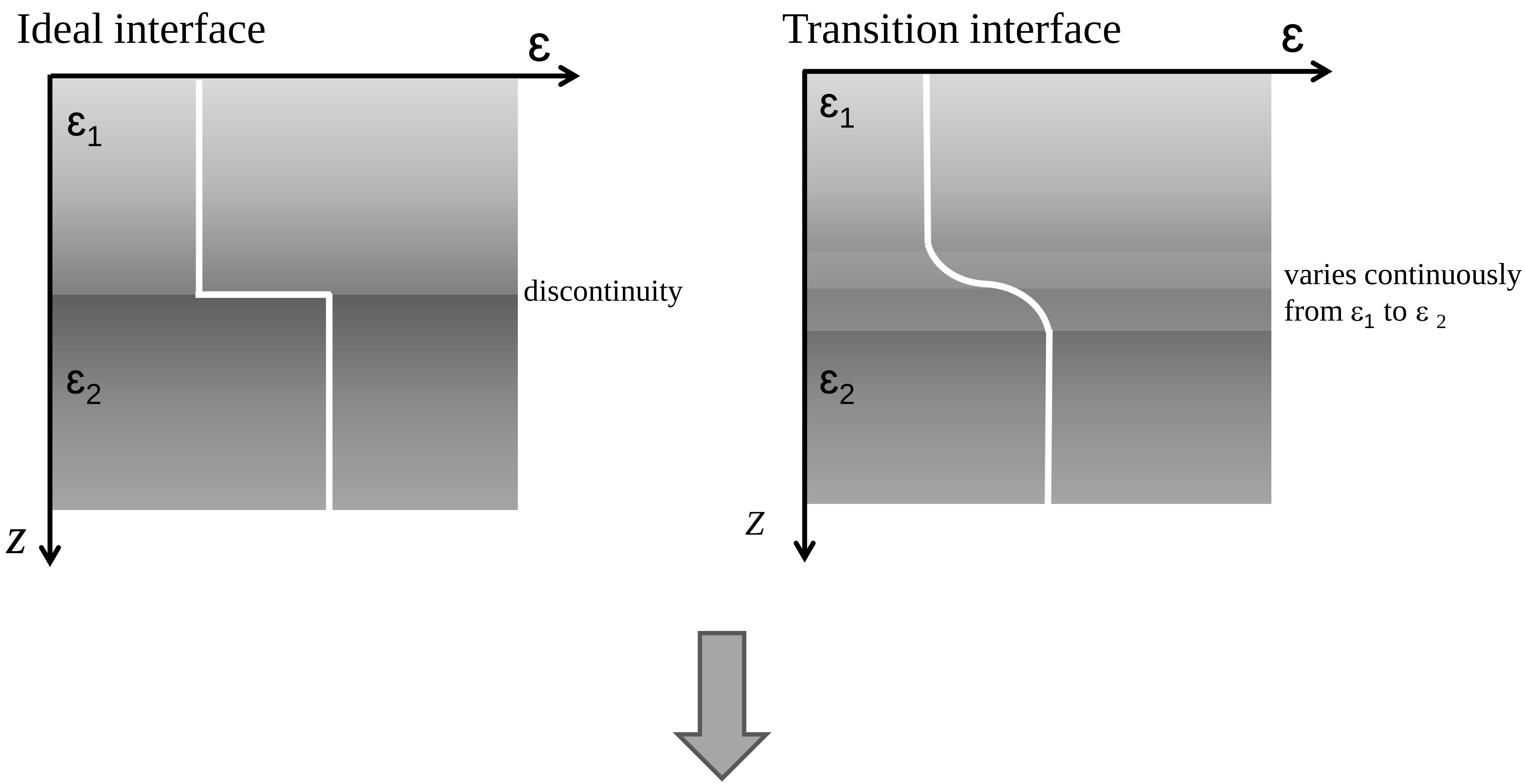
The theory of optics (P. Drude)

Section 2
Optical properties of bodies
Chapter 1
Theory of Light
Chapter 2
Transparent Isotropic Media

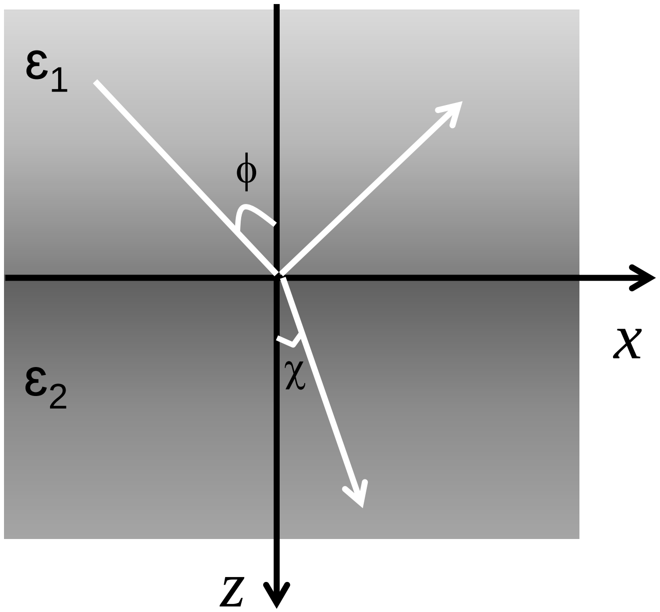
22OCT2011 SEOK, SANGJUN

At interface between two bulk phase



Elliptic polarization of the reflected light and the surface or transition layer

Reflection and Refraction at the boundary - between two transparent isotropic media



Plane wave is

$$X = A_x \cdot \cos\frac{2\pi}{T}\left(t - \frac{mx+ny+pz}{V}\right)$$

$$Y = A_y \cdot \cos\frac{2\pi}{T}\left(t - \frac{mx+ny+pz}{V}\right)$$

$$Z = A_z \cdot \cos\frac{2\pi}{T}\left(t - \frac{mx+ny+pz}{V}\right)$$

The direction cosines of the direction of propagation of the incident wave are …

$m = \sin\phi,\ n = 0,\ p = \cos\phi$ Let the *x*-*z* Incidence plane

Parallel to the *y*-axis

$$Y_e = E_s \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi + z\cos\phi}{V_1}\right)$$

Along the *x*- and *z*-axes, of the E_p must have

$$A_x = E_p \cdot \cos\phi,\ A_z = -E_P \cdot \sin\phi$$

$$X_e = E_p \cdot \cos\phi \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi + z\cos\phi}{V_1}\right)$$

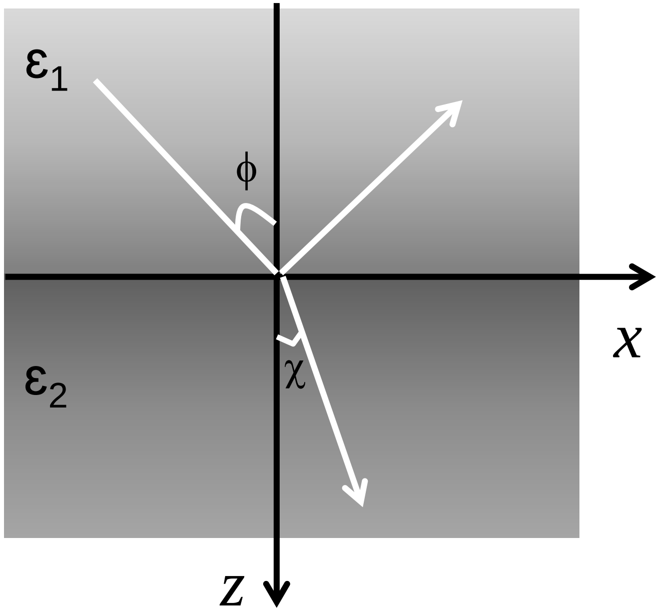
$$Z_e = -E_p \cdot \sin\phi \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi + z\cos\phi}{V_1}\right)$$

$$\alpha_e = -E_s \cdot \cos\phi \cdot \sqrt{\varepsilon_1}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi + z\cos\phi}{V_1}\right)$$

$$\beta_e = E_p \cdot \sqrt{\varepsilon_1}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi + z\cos\phi}{V_1}\right)$$

$$\gamma_e = E_s \cdot \sin\phi \cdot \sqrt{\varepsilon_1}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi + z\cos\phi}{V_1}\right)$$

Reflection and Refraction at the boundary - between two transparent isotropic media



The electric wave in the reflected and refracted

$$X_r = R_p \cdot \cos\phi' \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi' + z\cos\phi'}{V_1}\right)$$

$$Y_r = R_s \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi' + z\cos\phi'}{V_1}\right)$$

$$Z_r = -R_p \cdot \sin\phi' \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi' + z\cos\phi'}{V_1}\right)$$

$$X_2 = D_p \cdot \cos\chi \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\chi + z\cos\chi}{V_2}\right)$$

$$Y_2 = D_s \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\chi + z\cos\chi}{V_2}\right)$$

$$Z_2 = -D_p \cdot \sin\chi \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\chi + z\cos\chi}{V_2}\right)$$

The magnetic wave in the reflected and refracted

$$\alpha_r = -R_s \cdot \cos\phi' \cdot \sqrt{\varepsilon_1}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi' + z\cos\phi'}{V_1}\right)$$

$$\beta_r = R_p \cdot \sqrt{\varepsilon_1}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi' + z\cos\phi'}{V_1}\right)$$

$$\gamma_r = R_s \cdot \sin\phi' \cdot \sqrt{\varepsilon_1}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi' + z\cos\phi'}{V_1}\right)$$

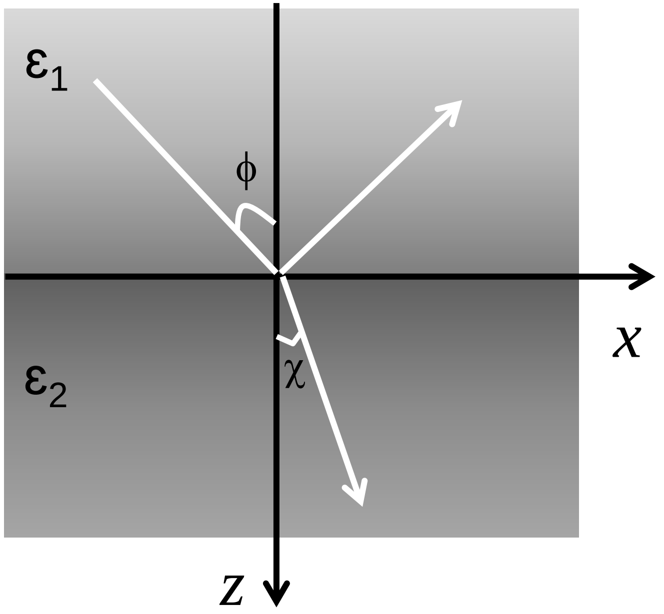
$$\alpha_2 = -D_s \cdot \cos\chi \cdot \sqrt{\varepsilon_2}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\chi + z\cos\chi}{V_2}\right)$$

$$\beta_2 = D_p \cdot \sqrt{\varepsilon_2}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\chi + z\cos\chi}{V_2}\right)$$

$$\gamma_2 = D_s \cdot \sin\chi \cdot \sqrt{\varepsilon_2}\cos\frac{2\pi}{T}\left(t - \frac{x\sin\chi + z\cos\chi}{V_2}\right)$$

Reflection and Refraction at the boundary - between two transparent isotropic media

ε



Infinitely thin B.D condition – for z=0

$$X_1 = X_2,\ Y_1 = Y_2,\ \alpha_1 = \alpha_2,\ \beta_1 = \beta_2$$

$$\varepsilon_1 Z_1 = \varepsilon_1 Z_2,\ \gamma_1 = \gamma_2$$

$$X_1 = X_e + X_{r,}\ \ \alpha_1 = \alpha_e + \alpha_r \ldots$$

$$(E_p - R_p)\cos\phi = D_p \cos\chi,$$

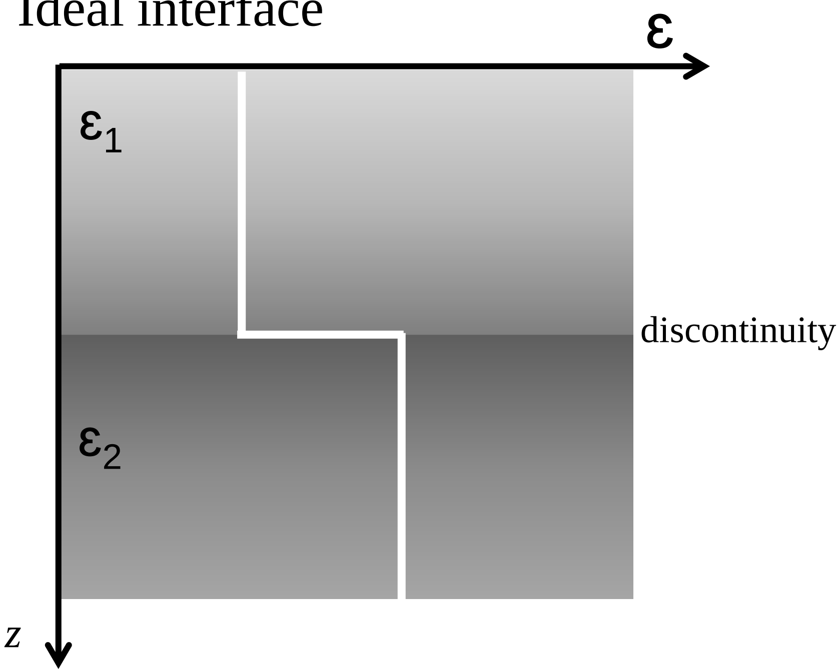
$$E_s + R_s = D_s,$$

$$(E_s - R_s)\sqrt{\varepsilon_1}\cos\phi = D_s\sqrt{\varepsilon_2}\cos\chi,$$

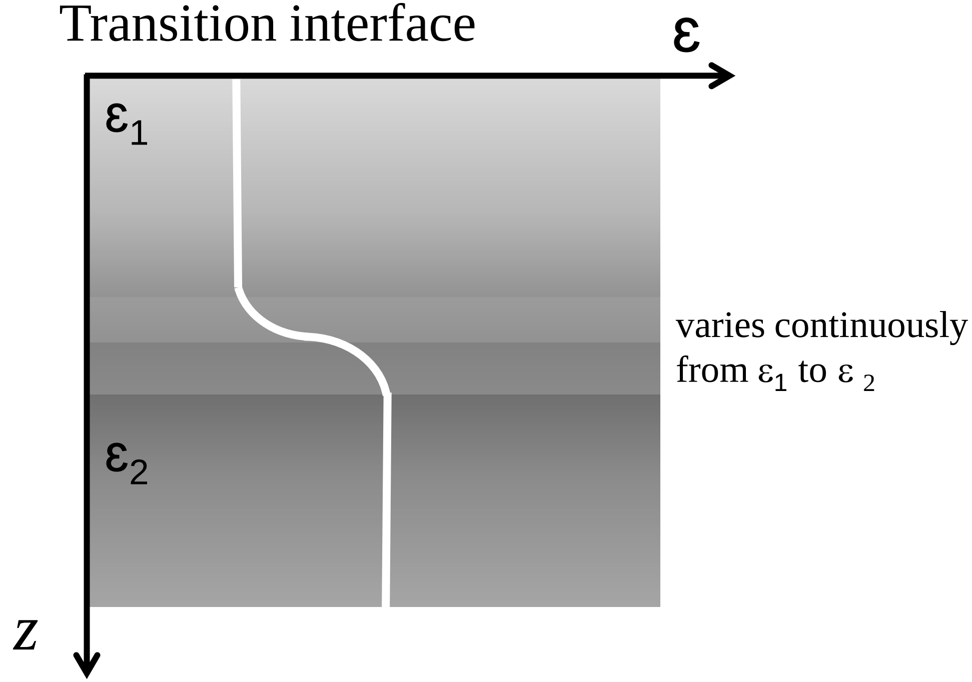
$$(E_p + R_s)\sqrt{\varepsilon_1} = D_p\sqrt{\varepsilon_2}$$

At interface between two bulk phase

Ideal interface



Transition interface



Elliptic polarization of the reflected light and the surface or transition layer

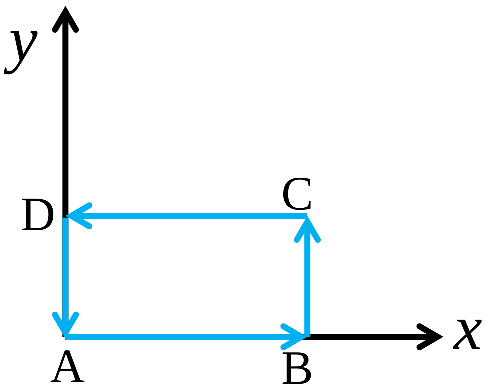
Definition of the electric and magnetic current in the electromagnetic system

In the electrostatic system the *current density i* ⇨ $i = \frac{de}{dt}$

If the cross-section is unity, *i* is equal to the current density *j*.

In the electromagnetic system the current i' is defined by the fact that it requires $4\pi i' = W$ units of work to carry unit magnetic pole once around the current.

Definition of the electric and magnetic current in the electromagnetic system



The whole work *W* done in moving a magnet pole m=+1 around the circuit from A through B, C, D, and back to A is

$$W = \alpha \cdot dx + \beta' \cdot dy - \alpha' \cdot dx - \beta \cdot dy$$

$$W = \left(\frac{\partial \beta}{\partial x} - \frac{\partial \alpha}{\partial y} \right) dxdy$$

$$\alpha' = \alpha + \frac{\partial \alpha}{\partial y} dy$$

$$\beta' = \beta + \frac{\partial \beta}{\partial x} dy$$

The current *i*´ this work is equal to $4\pi i' = 4\pi j' dxdy$

$$4\pi j'_z = \frac{\partial \beta}{\partial x} - \frac{\partial \alpha}{\partial y},$$

$$4\pi j'_x = \frac{\partial \gamma}{\partial y} - \frac{\partial \beta}{\partial z},$$

$$4\pi j'_y = \frac{\partial \alpha}{\partial z} - \frac{\partial \gamma}{\partial x}$$

electromagnetic current density *j*´
→ electrostatic current density *j*

$$\frac{4\pi}{c} j'_z = \frac{\partial \beta}{\partial x} - \frac{\partial \alpha}{\partial y},$$

$$\frac{4\pi}{c} j'_x = \frac{\partial \gamma}{\partial y} - \frac{\partial \beta}{\partial z},$$

$$\frac{4\pi}{c} j'_y = \frac{\partial \alpha}{\partial z} - \frac{\partial \gamma}{\partial x}$$

Magnetic current density

$$\frac{4\pi}{c} s_z = \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x},$$

$$\frac{4\pi}{c} s_x = \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y},$$

$$\frac{4\pi}{c} s_y = \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}$$

Definition of the electric and magnetic current in the electromagnetic system

magnetic current density

$$\frac{4\pi}{c} s_z = \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x},$$
$$\frac{4\pi}{c} s_x = \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y},$$
$$\frac{4\pi}{c} s_y = \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}$$

electric current density

$$\frac{4\pi}{c} j'_z = \frac{\partial \beta}{\partial x} - \frac{\partial \alpha}{\partial y},$$
$$\frac{4\pi}{c} j'_x = \frac{\partial \gamma}{\partial y} - \frac{\partial \beta}{\partial z},$$
$$\frac{4\pi}{c} j'_y = \frac{\partial \alpha}{\partial z} - \frac{\partial \gamma}{\partial x}$$

Electric or magnetic lines of force is numerically equal to the electric or magnetic force

$$4\pi j_x = \frac{\partial X}{\partial t},\ 4\pi j_y = \frac{\partial Y}{\partial t},\ 4\pi j_z = \frac{\partial Z}{\partial t},$$
$$4\pi s_x = \frac{\partial \alpha}{\partial t},\ 4\pi s_y = \frac{\partial \beta}{\partial t},\ 4\pi s_z = \frac{\partial \gamma}{\partial t}$$

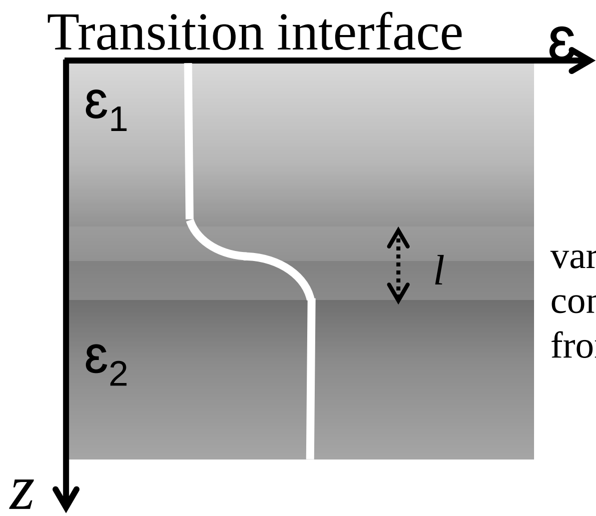
Isotropic Dielectrics

$$\frac{\varepsilon}{c}\frac{\partial X}{\partial t} = \frac{\partial \alpha}{\partial y} - \frac{\partial \beta}{\partial z},\ \frac{\varepsilon}{c}\frac{\partial Y}{\partial t} = \frac{\partial \alpha}{\partial z} - \frac{\partial \gamma}{\partial x},\ \frac{\varepsilon}{c}\frac{\partial Z}{\partial t} = \frac{\partial \beta}{\partial x} - \frac{\partial \alpha}{\partial y},$$
$$\frac{I}{c}\frac{\partial \alpha}{\partial t} = \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y},\ \frac{I}{c}\frac{\partial \beta}{\partial t} = \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z},\ \frac{I}{c}\frac{\partial \gamma}{\partial t} = \frac{\partial X}{\partial y} - \frac{\partial Z}{\partial x},$$

Elliptic Polarization of the Reflected light and surface or Transition layer

$$\frac{\varepsilon}{c}\frac{\partial X}{\partial t}=\frac{\partial \alpha}{\partial y}-\frac{\partial \beta}{\partial z},\quad \frac{\varepsilon}{c}\frac{\partial Y}{\partial t}=\frac{\partial \alpha}{\partial z}-\frac{\partial \gamma}{\partial x},\quad \frac{\varepsilon}{c}\frac{\partial Z}{\partial t}=\frac{\partial \beta}{\partial x}-\frac{\partial \alpha}{\partial y},$$

$$\frac{I}{c}\frac{\partial \alpha}{\partial t}=\frac{\partial Y}{\partial z}-\frac{\partial Z}{\partial y},\quad \frac{I}{c}\frac{\partial \beta}{\partial t}=\frac{\partial Z}{\partial x}-\frac{\partial X}{\partial z},\quad \frac{I}{c}\frac{\partial \gamma}{\partial t}=\frac{\partial X}{\partial y}-\frac{\partial Z}{\partial x},$$



varies continuously from ε_1 to ε_2

Multiplied by an element *dz* of the thickness of the transition layer

$$\frac{I}{c}\frac{\partial}{\partial t}\int_1^2 \alpha\cdot dz = Y_2 - Y_1,$$

$$\frac{I}{c}\frac{\partial}{\partial t}\int_1^2 \beta\cdot dz = \int_1^2 \frac{\partial Z}{\partial x}dz-(X_2-X_1)$$

α, β, and εZ approximately constant within the transition layer

$$\int \alpha\cdot dz=\alpha\int dz,\quad \int_1^2\frac{\partial Z}{\partial x}dz=\varepsilon_2\frac{\partial Z_2}{\partial x}\int_1^2\frac{dz}{\varepsilon}$$

Introducing the abbreviation

$$\int_1^2 dz = l,\quad \int_1^2 \varepsilon dz = p,\quad \int_1^2\frac{dz}{\varepsilon}=q$$

$$X_1 = X_2+\frac{l}{c}\frac{\partial \beta_2}{\partial t}-\varepsilon_2\frac{\partial Z_2}{\partial x}q,\quad Y_1=Y_2-\frac{l}{c}\frac{\partial \alpha_2}{\partial t}$$

Similarly…

$$\alpha_1=\alpha_2+l\frac{\partial \gamma_2}{\partial x}-\frac{p}{c}\frac{\partial Y_2}{\partial t},\quad \beta_1=\beta_2-\frac{p}{c}\frac{\partial X_2}{\partial t}$$

Elliptic Polarization of the Reflected light and surface or Transition layer

$$Y_e = E_s \cdot \cos\frac{2\pi}{T}\left(t - \frac{x\sin\phi + z\cos\phi}{V_1}\right)$$

Added to phase

$$Y_r = R_s \cos\left[\frac{2\pi}{T}\left(t - \frac{x\sin\phi' + z\cos\phi'}{V_1}\right) + \delta\right]$$

Y_r is the real part of the complex quantity

$$R_s \cdot e^{i\left[\frac{2\pi}{T}\left(t - \frac{x\sin\phi' + z\cos\phi'}{V_1}\right) + \delta\right]}$$

Writing now

$$R_s \cdot e^{i\delta} = \mathrm{R}_s$$

So Real amplitude E_s, E_p, R_s, R_p, etc., will be replaced by complex amplitude $\mathrm{E_s}$, $\mathrm{E_p}$, $\mathrm{R_s}$, $\mathrm{R_p}$, etc.

$X_1 = X_e + X_r$, $\alpha_1 = \alpha_e + \alpha_r$ for z=0

$$(\mathrm{E_p} - \mathrm{R_p})\cos\phi = \mathrm{D_p}\left[\cos\chi + i\frac{2\pi}{T}\left(\sqrt{\varepsilon_2}\frac{l}{c} - \frac{\sin^2\chi}{V_2}\varepsilon_2 q\right)\right]$$

$$(\mathrm{E_p} + \mathrm{R_p}) = \mathrm{D_s}\left[I + i\frac{2\pi}{T}\cos\chi\sqrt{\varepsilon_2}\frac{l}{c}\right]$$

$$(\mathrm{E_s} - \mathrm{R_s})\cos\phi = \mathrm{D_s}\left[\sqrt{\varepsilon_2}\cos\chi - i\frac{2\pi}{T}\left(\frac{\sin^2\chi}{V_2}\sqrt{\varepsilon_2}\, l - \frac{p}{c}\right)\right]$$

$$(\mathrm{E_p} + \mathrm{R_p})\sqrt{\varepsilon 1} = \mathrm{D}_p\left[\sqrt{\varepsilon_2} + i\frac{2\pi}{T}\cos\chi\frac{p}{c}\right]$$

Elliptic Polarization of the Reflected light and surface or Transition layer

$$\frac{R_P}{E_P}=\frac{\cos\phi\sqrt{\varepsilon_2}-\cos\chi\sqrt{\varepsilon_1}+i\frac{2\pi}{\lambda}\left[p\cos\phi\cos\chi-(l-q\varepsilon_2\sin^2\chi)\sqrt{\varepsilon_1\varepsilon_2}\right]}{\cos\phi\sqrt{\varepsilon_2}+\cos\chi\sqrt{\varepsilon_1}+i\frac{2\pi}{\lambda}\left[p\cos\phi\cos\chi+(l-q\varepsilon_2\sin^2\chi)\sqrt{\varepsilon_1\varepsilon_2}\right]}$$

$$\frac{R_s}{E_s}=\frac{\cos\phi\sqrt{\varepsilon_2}-\cos\chi\sqrt{\varepsilon_1}+i\frac{2\pi}{\lambda}\left[l\cos\phi\cos\chi\sqrt{\varepsilon_1\varepsilon_2}-p+l\varepsilon_2\sin^2\chi)\right]}{\cos\phi\sqrt{\varepsilon_2}+\cos\chi\sqrt{\varepsilon_1}+i\frac{2\pi}{\lambda}\left[l\cos\phi\cos\chi\sqrt{\varepsilon_1\varepsilon_2}+p-l\varepsilon_2\sin^2\chi)\right]}$$

$$\frac{R_P}{E_P}=\frac{\cos\phi\sqrt{\varepsilon_2}-\cos\chi\sqrt{\varepsilon_1}}{\cos\phi\sqrt{\varepsilon_2}+\cos\chi\sqrt{\varepsilon_1}}\left\{I+i\frac{4\pi}{\lambda}\cos\phi\sqrt{\varepsilon_1}\frac{-p\cos^2\chi-l\varepsilon_2^2\sin^2\chi}{\varepsilon_2\cos^2\phi-\varepsilon_1\cos^2\chi}\right\}...1$$

$$\frac{R_S}{E_S}=\frac{\cos\phi\sqrt{\varepsilon_1}-\cos\chi\sqrt{\varepsilon_2}}{\cos\phi\sqrt{\varepsilon_1}+\cos\chi\sqrt{\varepsilon_2}}\left\{I+i\frac{4\pi}{\lambda}\cos\phi\sqrt{\varepsilon_1}\frac{l\varepsilon_2-p}{\varepsilon_1\cos^2\phi-\varepsilon_2\cos^2\chi}\right\}...2$$

Snell's law

$$\sqrt{\varepsilon_1}\sin\phi=\sqrt{\varepsilon_2}\sin\chi$$

$$\varepsilon_1\cos^2\phi-\varepsilon_2\cos^2\chi=\varepsilon_1-\varepsilon_2$$

$$\varepsilon_2\cos^2\phi-\varepsilon_1\cos^2\chi=\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_2}(\varepsilon_1\sin^2\phi-\varepsilon_2\cos^2\phi)$$

Assumed by 45° pol.

$$\frac{R_P}{R_S}=-\frac{\cos(\phi+\chi)}{\cos(\phi-\chi)}\left\{I+i\frac{4\pi}{\lambda}\frac{\varepsilon_2\sqrt{\varepsilon_1}}{\varepsilon_1-\varepsilon_2}\cdot\frac{\cos\phi\sin^2\phi}{\varepsilon_1\sin^2\phi-\varepsilon_2\cos^2\phi}\eta\right\}$$

Elliptic Polarization of the Reflected light and surface or Transition layer

$$\frac{\mathrm{R_P}}{\mathrm{R_S}} = -\frac{\cos(\phi+\chi)}{\cos(\phi-\chi)}\left\{\mathrm{I} + i\frac{4\pi}{\lambda}\frac{\varepsilon_2\sqrt{\varepsilon_1}}{\varepsilon_1-\varepsilon_2}\cdot\frac{\cos\phi\sin^2\phi}{\varepsilon_1\sin^2\phi-\varepsilon_2\cos^2\phi}\eta\right\}$$

η is an abbreviation

$$\eta = p - l(\varepsilon_1+\varepsilon_2) + q\varepsilon_1\varepsilon_2$$

At the near Brewster angle

$$\frac{\mathrm{R_P}}{\mathrm{R_S}} = i\frac{\pi}{\lambda}\frac{\sqrt{\varepsilon_1+\varepsilon_2}}{\varepsilon_1-\varepsilon_2}\eta$$

$$\mathrm{R_P} = R_p\cdot e^{i\delta_p}, \mathrm{R_P} = R_s\cdot e^{i\delta_s} \qquad \frac{\mathrm{R_P}}{\mathrm{R_S}} = \frac{R_P}{R_S}e^{i(\delta_P-\delta_s)} = \rho\cdot e^{i\Delta}$$

$$\bar{\rho} = i\frac{\pi}{\lambda}\frac{\sqrt{\varepsilon_1+\varepsilon_2}}{\varepsilon_1-\varepsilon_2}\eta,\ \Delta=\pi/2$$

$$\bar{\rho} = i\frac{\pi}{\lambda}\frac{\sqrt{\varepsilon_1+\varepsilon_2}}{\varepsilon_1-\varepsilon_2}\cdot\int\frac{(\varepsilon-\varepsilon_1)(\varepsilon-\varepsilon_2)}{\varepsilon}dz$$

An ellipsometric study of the surface freezing of liquid alkanes

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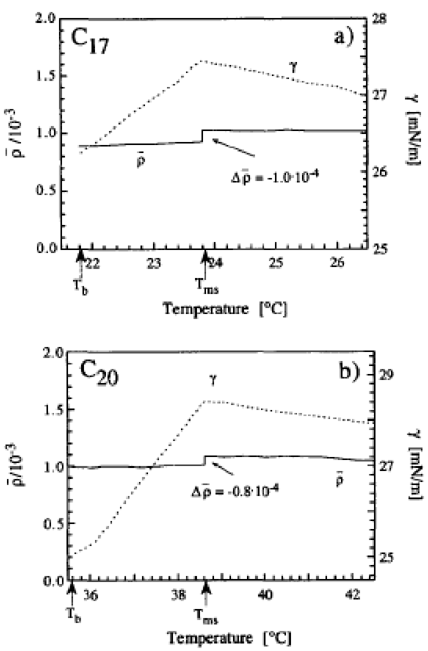
Abstract

The surface freezing of alkane/air interfaces was investigated by ellipsometry and surface tension measurements. It is observed as a small jump in the ellipsometric signal. This shift can be explained by the transition from an isotropic liquid surface to a well-ordered monolayer only through a fortituous cancellation of layering and anisotropy effects. Therefore, an alternative model is discussed which interprets the surface freezing as a transition from a nematic-like molecular ordering at the interface above the surface melting temperature to a smectic-like ordering below.



$$\frac{R_P}{R_S} = \frac{R_P}{R_S} e^{i(\delta_P - \delta_s)} = \rho \cdot e^{i\Delta}$$

$$\eta = \frac{\lambda}{\pi} \frac{(\epsilon_1 - \epsilon_2)}{\sqrt{\epsilon_1 + \epsilon_2}} \bar{\rho}.$$