Fluctuation properties of chaotic light

The quantum theory of light
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❖ *Two types of light source*

- Chaotic light

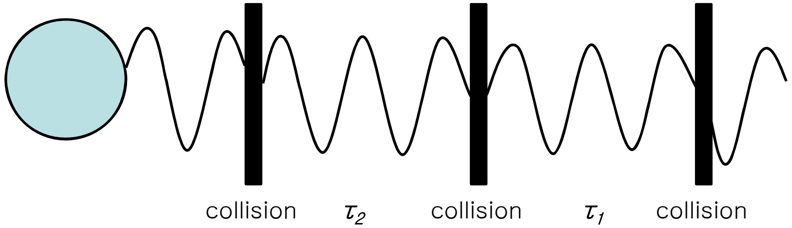
  (thermal cavity, filament lamp)

  The different atoms are excited by an electrical discharge and emit their radiation independently of one another.

  The shape of an emission line is determined by the statistical spread in atomic velocities and the random occurrence of collisions.

- Laser

❖ Model of collision-broadened light source

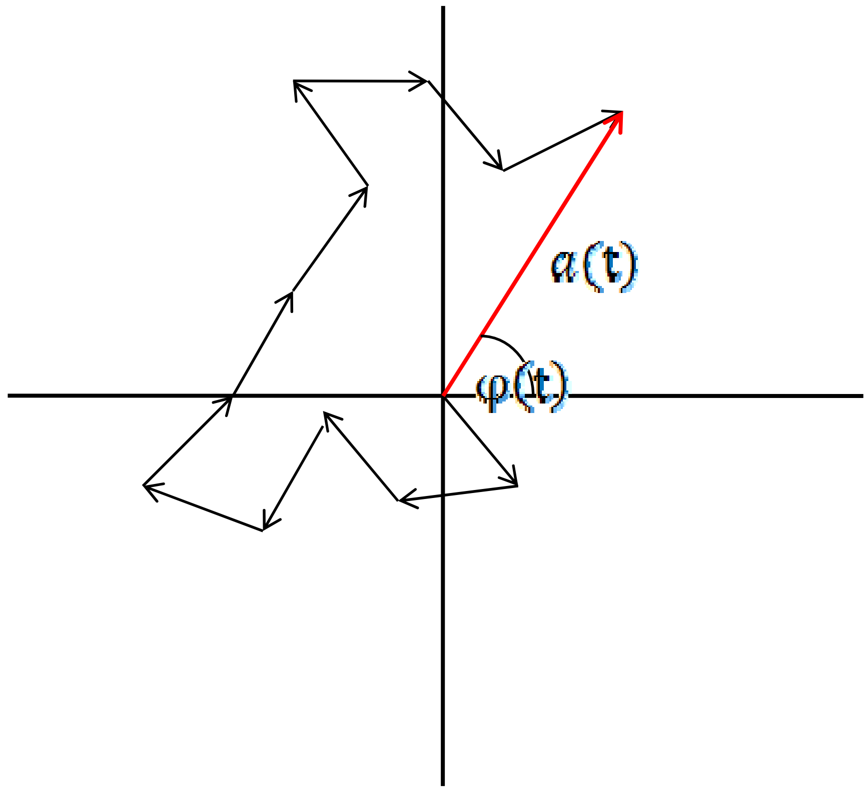


$$E(t) = E_0 \exp\{-i\omega_0 t + i\phi(t)\}$$

The phase $\phi(t)$ remains constant during periods of free flight but changes abruptly each time a collision occurs. The amplitude E_0 And frequency ω_0 are the same for any period. If there is a large number v of such atoms, the total electric field amplitude is

$$\begin{aligned} E(t) &= E_1(t) + E_2(t) + \cdots + E_v(t) \\ &= E_0 \exp(-i\omega_0 t)\left\{\exp\left(i\phi_1(t)\right) + \exp\left(i\phi_2(t)\right) + \cdots + \exp\left(i\phi_v(t)\right)\right\} \\ &= E_0 \exp(-i\omega_0 t) a(t) \exp\left(i\varphi(t)\right) \end{aligned}$$

❖ Model of collision-broadened light source



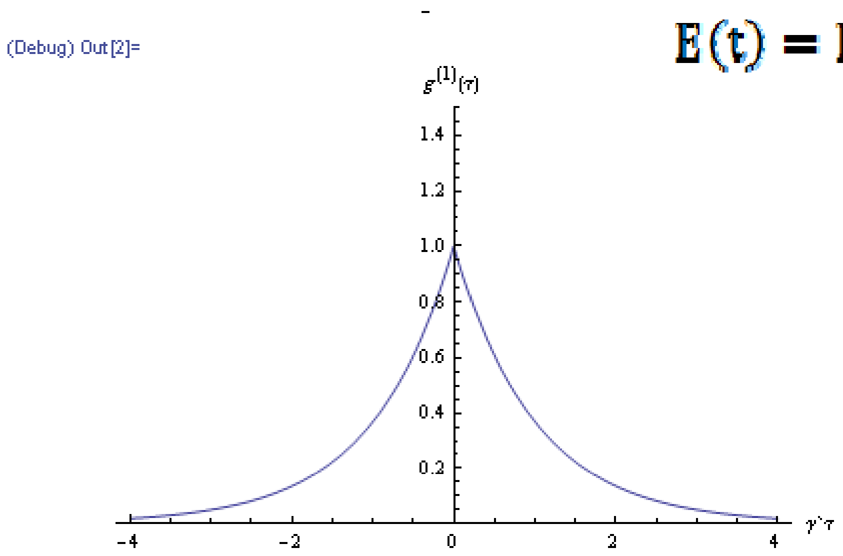
Argand diagram to show the amplitude and phase of the resultant vector formed by a large number of unit vectors, each of which has a randomly chosen phase angle.

❖ Degree of first-order coherence

$$g^{(1)}(\tau) = \frac{\langle E^*(t)\, E(t+\tau) \rangle}{\langle E^*(t) E(t) \rangle}$$

$$g^{(1)}(z_1, t_1, z_2, t_2) \equiv g^{(1)}(\tau) = e^{-i\omega_0\tau - \gamma`|\tau|}$$

$$E(t) = E_0 \exp\{-i\omega_0 t + i\phi(t)\}$$

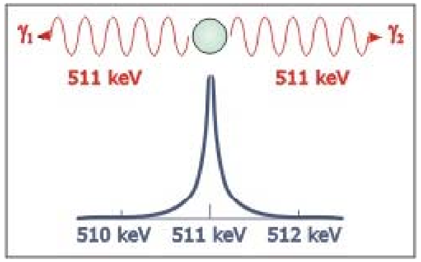


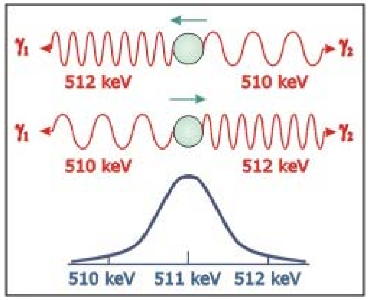
The modulus of the degree of first-order coherence for chaotic light of linewidth parameter $\gamma`$.

❖ Degree of first-order coherence

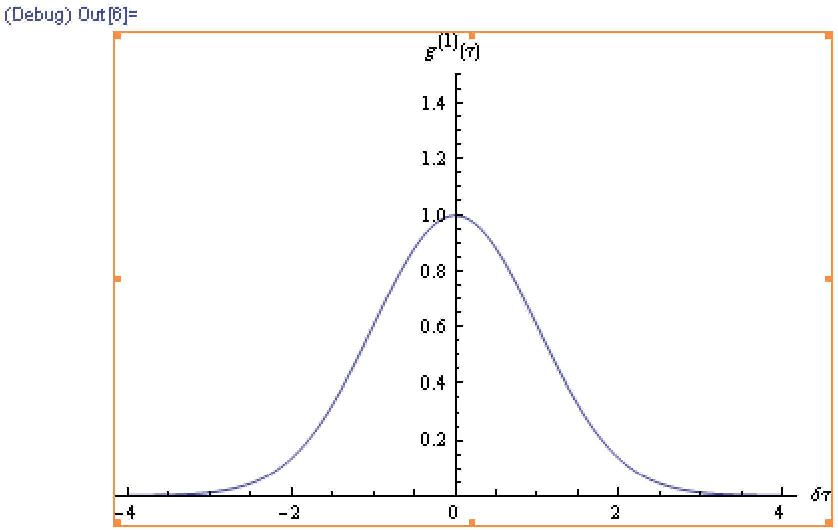
Doppler broadening

$$E(t) = E_0 \sum_{i=1}^{\nu} \exp(-i w_i t + i\phi_i)$$





$$g^{(1)}(\tau) = e^{-i\omega_0\tau - \frac{1}{2}\delta^2\tau^2}$$



The modulus of the degree of first-order coherence for chaotic light of Gaussian frequency distribution with root-mean-square width δ.

❖ *Intensity fluctuations of chaotic light*

- The second main topic -> direct measurement intensity fluctuations
- We consider the statistical properties of the intensity fluctuations in chaotic light.

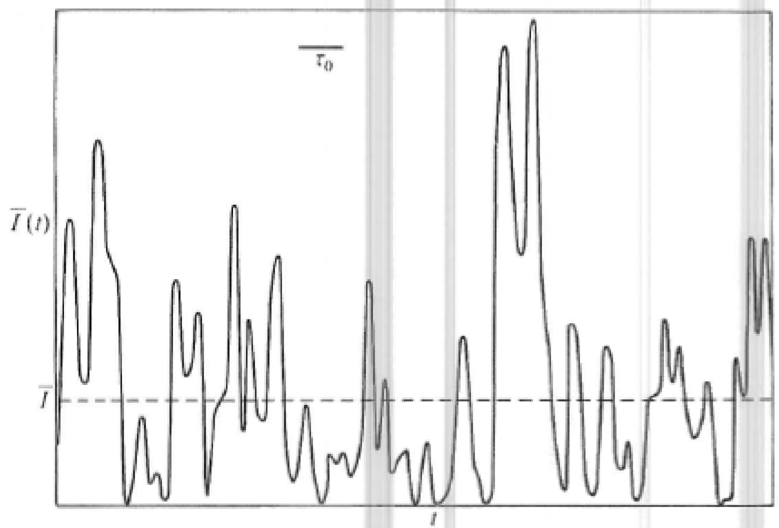


FIG. 3.4. Time dependence of the cycle-averaged intensity for a chaotic light beam, obtained from a computer simulation. The mean time τ_0 between collisions has the magnitude indicated. The dashed line shows the mean value of the intensity averaged over a time long compared to τ_0. (Computation carried out by Mrs S. Sussmann.)

❖ Intensity fluctuations of chaotic light

- Suppose initially there is available an ideal detector, with response time much shorter than the coherence time τ_c.
- Intensity $\bar{I}(t)$ taken over a period of time very much longer than τ_c.

- Long time average intensity is

$$\bar{I} \equiv \langle \bar{I}(t) \rangle = \frac{1}{2}\epsilon_0 c E_0^2 \langle |\exp(i\phi_1(t)) + \exp(i\phi_2(t)) + \cdots + \exp(i\phi_\nu(t))|^2 \rangle$$
$$= \frac{1}{2}\epsilon_0 c E_0^2 \nu$$

❖ Intensity fluctuations of chaotic light

- Mean square intensity is

$$\langle \bar{I}(t)^2 \rangle = \frac{1}{4}\epsilon_0^2 c^2 E_0^4 \langle |\exp(i\phi_1(t)) + \exp(i\phi_2(t)) + \cdots + \exp(i\phi_\nu(t))|^4 \rangle$$

- These terms give

$$\langle \bar{I}(t)^2 \rangle = \frac{1}{4}\epsilon_0^2 c^2 E_0^4 \{\nu + 2\nu(\nu - 1)\}$$

- Compare long time average intensity and mean square intensity, then the mean-square intensity is

$$\langle \bar{I}(t)^2 \rangle = \left(2 - \frac{1}{\nu}\right)\bar{I}^2$$

❖ *Intensity fluctuations of chaotic light*

- The number ν of radiating atoms is normally very large.

$$\langle \bar{I}(t)^2 \rangle = 2\bar{I}^2 \quad (\nu \gg 1)$$

- The root mean square deviation of the cycle-averaged intensity is

$$\left(\langle \bar{I}(t)^2 \rangle - \langle \bar{I}(t) \rangle^2 \right)^{\frac{1}{2}} = \bar{I}$$

- The size of fluctuation is thus equal to the average intensity.

❖ Degree of second order coherence

- We now consider two time measurements in which a series of pairs of intensity readings are taken with a fixed time-delay τ.

- We here consider the theory of the "corrlation function"

$$g^{(2)}(\tau) = \frac{\langle \bar{I}(t)\bar{I}(t+\tau)\rangle}{\bar{I}^{\,2}} = \frac{\langle E^{*}(t)E^{*}(t+\tau)E(t+\tau)E(t)\rangle}{\langle E^{*}(t)E(t)\rangle^{2}}$$

$$g^{(2)}(-\tau) = g^{(2)}(\tau)$$ symmetry

❖ Degree of second order coherence

$$2\bar{I}(t_1)\bar{I}(t_2) \leq \bar{I}(t_1)^2 + \bar{I}(t_2)^2$$

By applying this inequality to the cross-terms, it is easy to show that

$$\left\{\frac{\bar{I}(t_1) + \bar{I}(t_2) + \cdots + \bar{I}(t_N)}{N}\right\}^2 \leq \frac{\bar{I}(t_1)^2 + \bar{I}(t_2)^2 + \cdots + \bar{I}(t_N)^2}{N}$$

for the results of N measurements of the intensity. Thus in the correlation function notation.

$$\bar{I}^{\,2} \equiv \langle \bar{I}(t) \rangle^2 \leq \langle \bar{I}(t)^2 \rangle$$

❖ Degree of second order coherence

- And the zero time-delay degree of second -order coherence satisfies

$$g^{(2)}(0) \geq 1$$

- It is not possible to establish any upper limit

$$\infty \geq g^{(2)}(0) \geq 1$$

- The above proof cannot be extended to finite time delays, and the only restriction then results from the essentially positive nature of the intensity,

$$\infty \geq g^{(2)}(\tau) \geq 0 \qquad \tau \neq 0$$

❖ Degree of second order coherence

$$\left[\bar{I}(t_1)\bar{I}(t_1+\tau)+\cdots+\bar{I}(t_N)\bar{I}(t_N+\tau)\right]^2$$
$$\leq\left[\bar{I}(t_1)^2+\cdots+\bar{I}(t_N)^2\right]\left[\bar{I}(t_1+\tau)^2+\cdots+\bar{I}(t_N+\tau)^2\right]$$

- The two summations on the right are equal for a sufficiently long and numerous series of measurements, and the square root then produces the result

$$\langle\bar{I}(t)\bar{I}(t+\tau)\rangle\leq\langle\bar{I}(t)^2\rangle$$

$$g^{(2)}(\tau)\leq g^{(2)}(0)$$

❖ Second order coherence of chaotic light

- The statistical properties of chaotic light produce beam intensities that are uncorrelated after time separations long compared to the coherence time τ_c .
- The degree of second-order coherence $g^{(2)}(\tau)$ thus has a limiting value

$$g^{(2)}(\tau) \rightarrow 1 \quad \tau \gg \tau_c$$

❖ Second order coherence of chaotic light

- Independent contributions from the different radiating atoms I

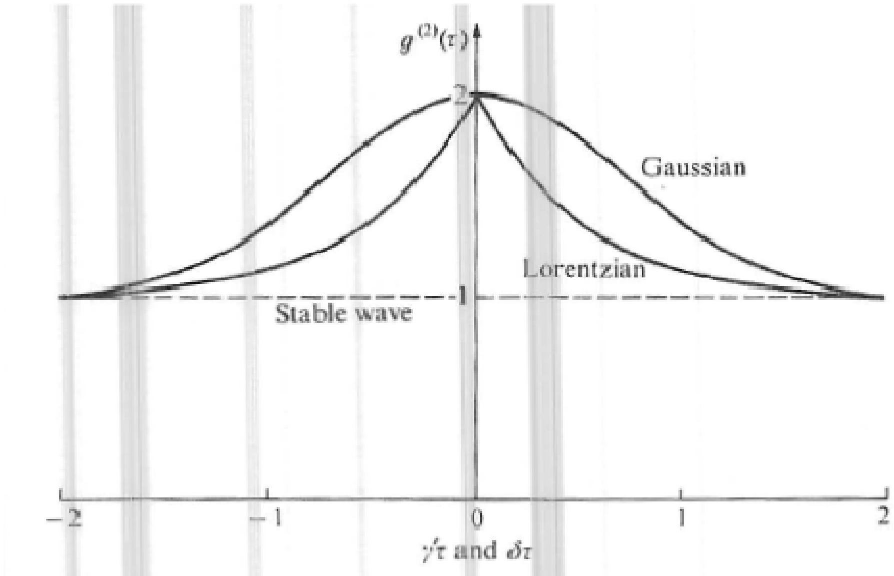
$$E(t) = \sum_i E_i(t)$$

- If the number v of radiating atoms is assumed to be large, and with the definitions of the degrees of first and second order coherence, give

$$g^{(2)}(\tau) \rightarrow 1 + \left|g^{(1)}(\tau)\right|^2 \quad (\nu \gg)$$

- This important relation holds for all varieties of chaotic light.

❖ Second order coherence of chaotic light



The degree of second order coherence for the chaotic light of Lorentzian frequency distribution and Gaussian frequency distribution.