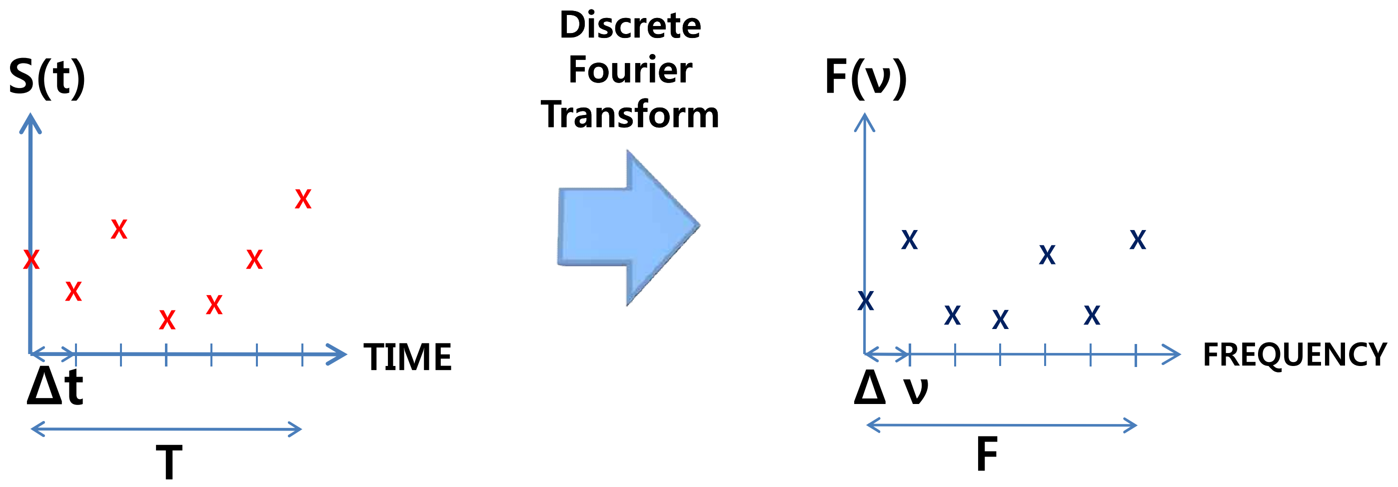
Discrete Fourier Transform (DFT)



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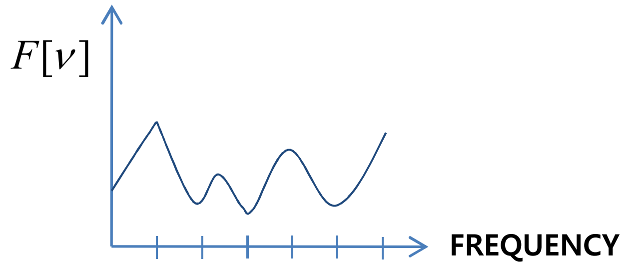
Discrete Fourier Transform

The Discrete Fourier Transform is the equivalent of the continuous Fourier Transform for signals known only at N instant separated by sample times T.

✓ **Fourier Transform** :

$$F(\nu) = \int_{-\infty}^{\infty} S(t) e^{i2\pi\nu t} dt$$

$S(t),\ F(\nu)$: continuous signal

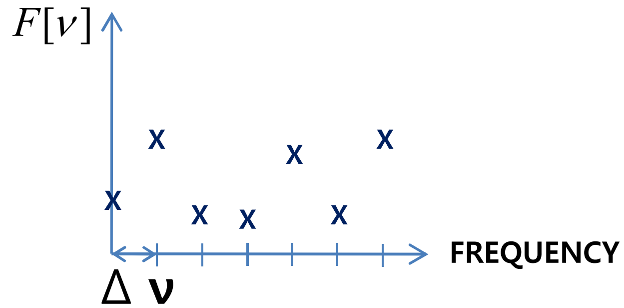


✓ **Discrete Fourier Transform** :

$$F(\nu_i) = \sum_{n=0}^{N-1} S(t_n) e^{i2\pi\nu_i t_n}$$

$S(t_j)$: j^{th} signal

n=0,1,2,$\cdots$,N-1



$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ \vdots \\ F(N-1) \end{bmatrix} = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & e^{i2\pi\nu_1 t_1} & e^{i2\pi\nu_1 t_2} & \cdots & e^{i2\pi\nu_1 t_{N-1}} \\ 1 & e^{i2\pi\nu_2 t_1} & e^{i2\pi\nu_2 t_2} & \cdots & e^{i2\pi\nu_2 t_{N-1}} \\ \vdots & & & & \\ 1 & e^{i2\pi\nu_{N-1} t_1} & e^{i2\pi\nu_{N-1} t_2} & \cdots & e^{i2\pi\nu_{N-1} t_{N-1}} \end{pmatrix} \begin{bmatrix} S(0) \\ S(1) \\ S(2) \\ \vdots \\ S(N-1) \end{bmatrix}$$

Determine △ν (interval frequency)

① $$\sum_{j=0}^{N-1} e^{i\omega_j t_i} \times (e^{i\omega_j t_k})^* = \delta_{ik} N \quad ; \omega_j = \frac{1}{N-1} j\omega_{tot}, \ \omega = 2\pi\nu$$

$$\sum_{j=0}^{N-1} r^j = \frac{1-r^N}{1-r}$$

here.

$$r^N = (e^{i(t_i - t_k)\frac{1}{N-1}\omega_{tot}})^N$$

- $i = k$

$$\delta_{ik} N = 1$$

- $i \neq k$

$$\delta_{ik} N = 0$$

② $$\sum_{j=0}^{N-1} e^{i(t_i - t_k)\frac{1}{N-1} j\omega_{tot}} = \frac{1 - e^{i(\frac{\omega_{tot}}{N-1})(t_i - t_k)N}}{1 - e^{i(\frac{\omega_{tot}}{N-1})(t_i - t_k)}} = 0 \ ;$$

$$\omega_{tot}(t_i - t_k) = 0, 2\pi \qquad \text{for } t_i = \Delta t \times (i-1), \ t_k = \Delta t \times (k-1)$$

$$\omega_{tot} \Delta t_{(i-k)} = 0, 2\pi$$

$$\omega_{tot} = \frac{2\pi}{\Delta t}, \quad \nu_{tot} = \frac{1}{\Delta t}$$

$$\nu_{tot} = N \cdot \Delta\nu = \frac{1}{\Delta t}, \quad \Delta\nu = \frac{1}{T_{tot}}$$

- $\Delta\nu$: interval frequency
- ν_{tot} : total frequency
- Δt : interval time
- T_{tot} : total time

✓ **decrese △t => Increase total ν_{tot}**

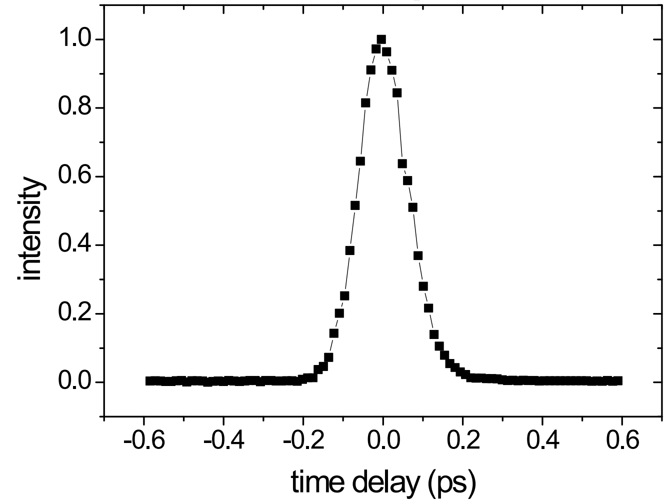
✓ **Increase total T_{tot} => decres △ν (spectral density resolution increase)**

Discrete Fourier Transform of Gaussian function

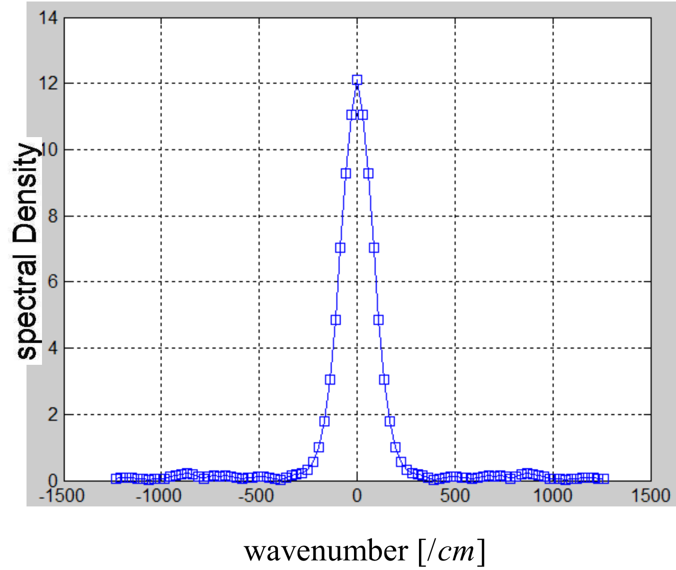
$$\int_{-\infty}^{\infty} e^{-\frac{t^2}{a^2}} \cdot e^{i\omega t} dt = a\sqrt{\pi} e^{-\frac{w^2 a^2}{4}}$$

①

Δt = 0.0132 [ps] , N=90

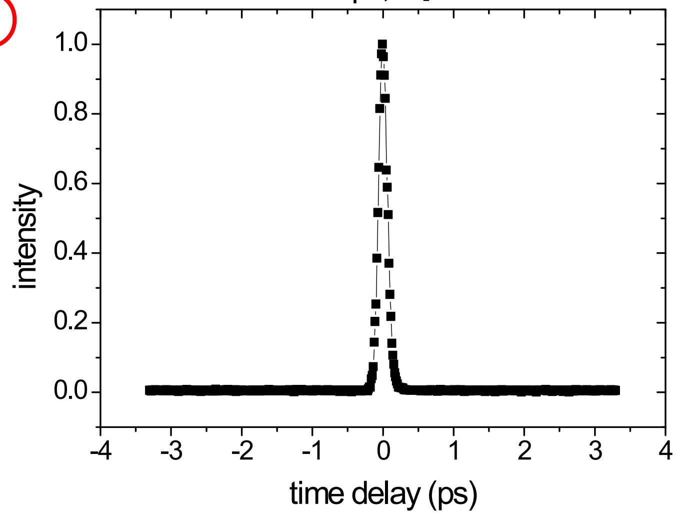


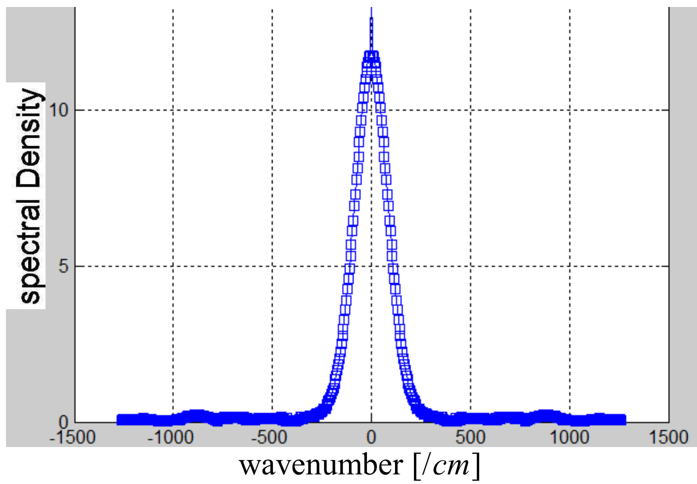
Discrete Fourier Transform



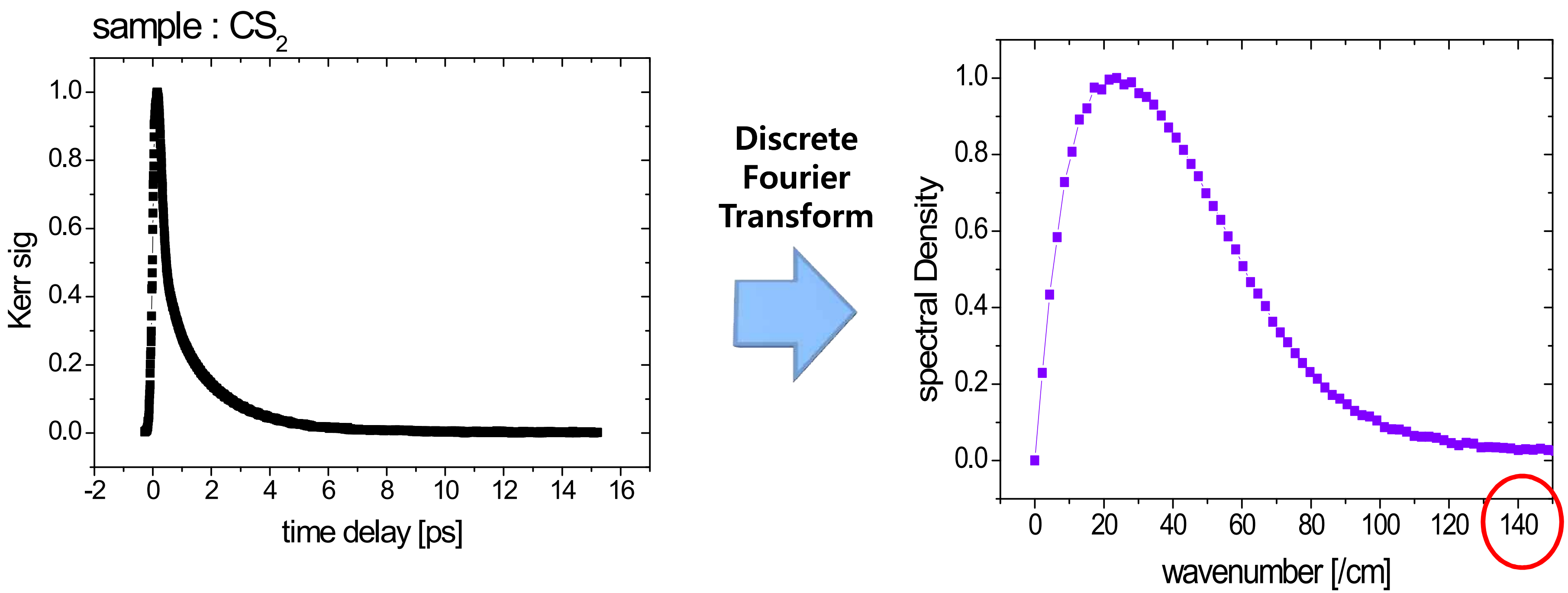
②

Δt = 0.0132 [ps] , N=501





Discrete Fourier Transform of Kerr Signal



$$\Delta t = 0.0066[ps]$$

$$N = 2347$$

$$T_{tot} = 15.4902[ps]$$

$$\nu_{tot} = \frac{1}{\Delta t} = 150[THz]$$

$$\lambda_{tot} = \nu_{tot} \times \frac{1}{c} = 5050[/cm]$$