The thermodynamic meaning of negative entropy

Lídia del Rio, Johan Åberg, Renato Renner,
Oscar Dahlsten & Vlatko Vedral

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Introduction

- Landauer's principle states that the erasure of data stored in a system has an inherent work cost and therefore dissipates heat
- the more an observer knows about the system, the less it costs to erase it

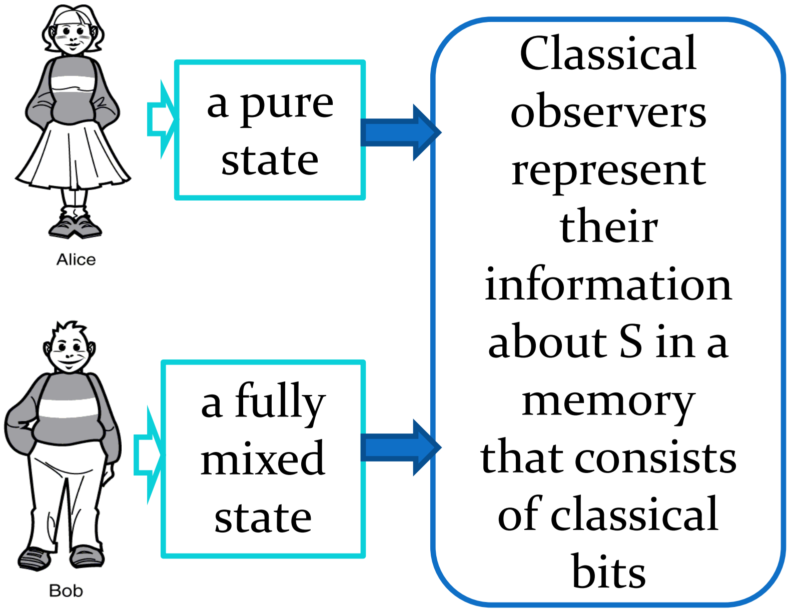
- ‘Erasure’ of a system is defined as taking it to a pre-defined pure state, |0>
- The energy dissipated to erase a system, S

$$W(S) = H(S)kT\ln(2) \qquad (1)$$

$$H(S) = -\mathrm{Tr}[\rho \log_2(\rho)]$$

the von Neumann entropy

the Boltzmann constant



In equation (1) W(S) - the 'cost of erasure'

Denote (1) by W(S|C)- the 'cost of erasure for observer C'

$$W(S|C) = H(S|C)kT\ln(2) \qquad (2)$$

The observer C is assumed to be classical

Quasimodo

He has a memory that includes n qubits, each maximally entangled with a particle of S

$$W(S|Q) = H(S|Q)kT\ln(2) \qquad (3)$$

$H(S|Q)=H(SQ)-H(Q)$

This work cost is optimal, under the assumption that Landauer's principle holds for a classical observer

Methods Summary

Figure 1 | Erasure in quantum computation.

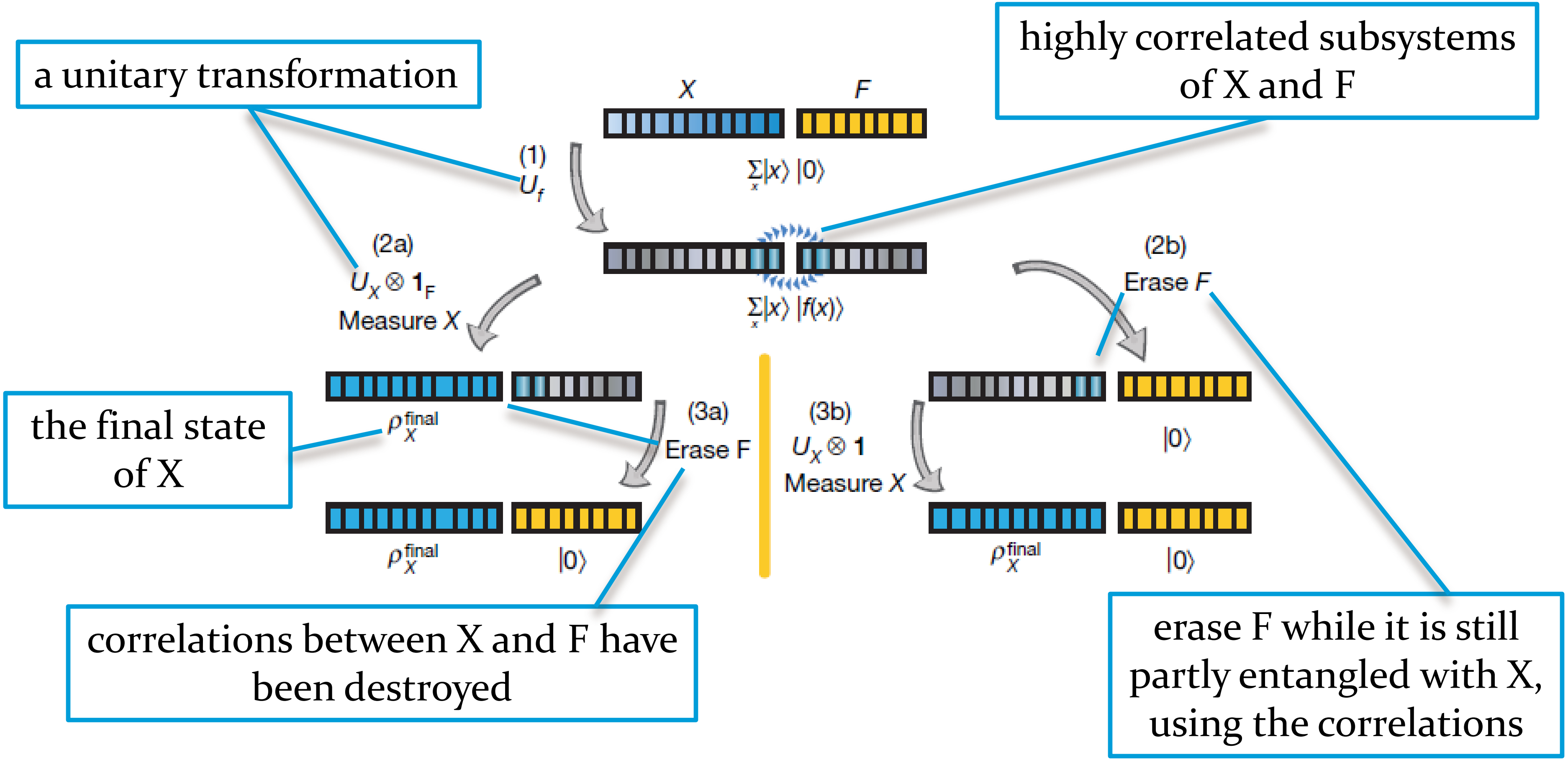
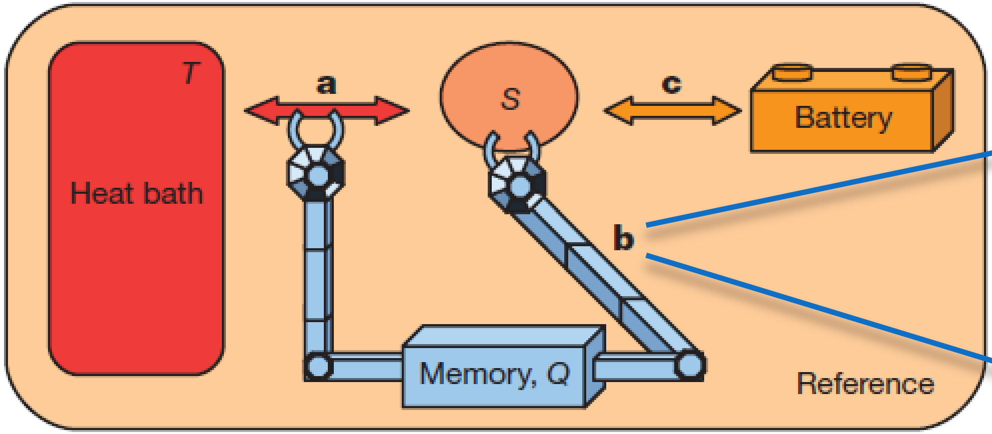


Figure 2 | Erasure setting.

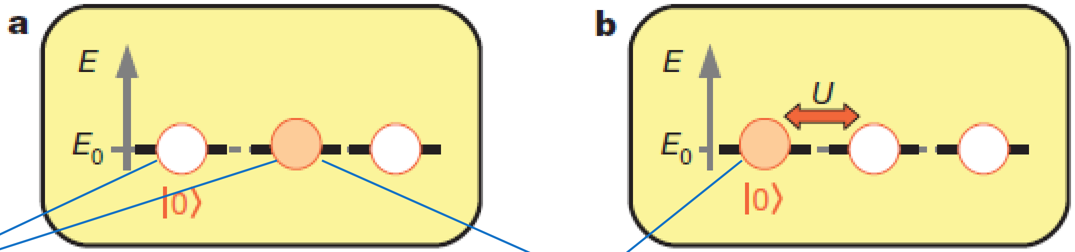


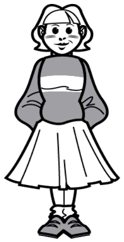
(1) applying unitary operations to those systems

(2) raising or lowering any energy levels of their Hamiltonian

Figure 3 | Erasure of a pure state.

The work required for erasure may be negative for an observer with a quantum memory: the process results in a net gain of work (from eq. 3)



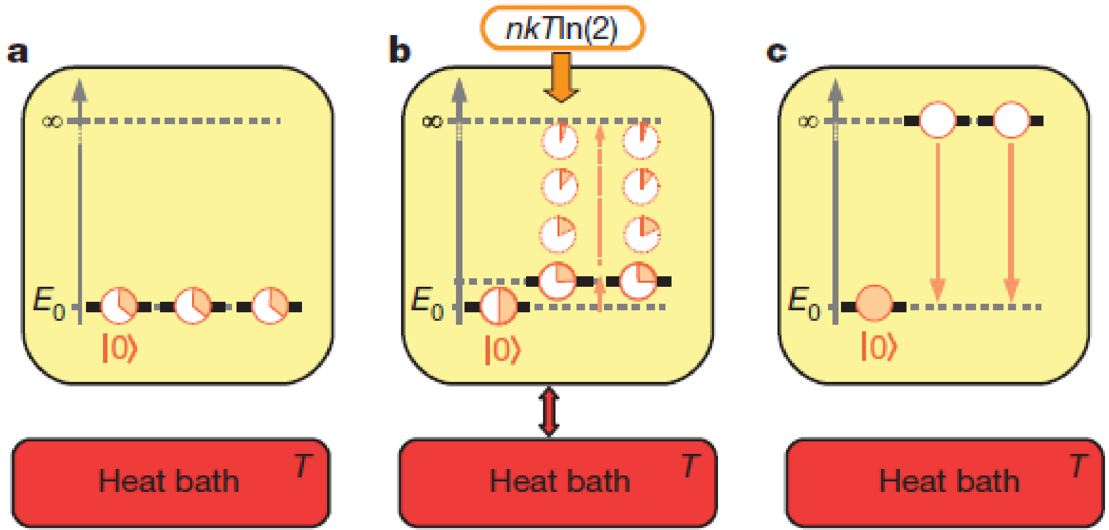


Alice

The circles represent the energy eigenstates of system S

A filled circle means that the system can be found in that state with certainty

Figure 4 | Erasure of a fully mixed state and work extraction.



Bob has maximal uncertainty: H(S|B)=n.
The work cost of this process is *nkTln(2)*.

Quasimodo

His memory contains n qubits maximally entangled with S

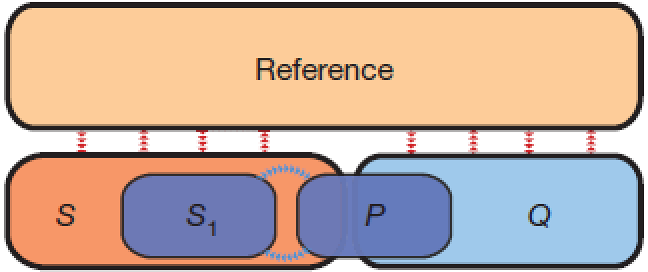
They call this part of his memory Q1 and denote the entangled state |SQ1>

The rest of his memory, Q2, is correlated with a reference system, R, in state |Q2R>

To erase S, Quasimodo combines two elementary procedures:

- The erasure process used by Bob
- Its reverse

Figure 5 | General erasure procedure.



$$\log_2(|S_1|) \approx [\log_2(|S|) - H(S|Q)]/2$$

$$2\log_2(|S_1|)kT\ln(2).$$

$$\log_2(|S|)kT\ln(2)$$

$$H(S|Q)kT\ln(2)$$

Summarize

- the erasure can be optimized if information stored in other parts of the memory is used
- the results suggest that discord can quantify the difference between the respective work costs of erasure using quantum and classical memories
- infer that the conditional entropy $H(S|Q)$ cannot decrease under local operations on Q, which is a fundamental result in information theory known as the data processing inequality