Negative Lateral Shift of a Light Beam Transmitted through a Dielectric Slab and Interaction of Boundary Effects

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It is found that when a light beam travels through a slab of optically denser dielectric medium in air, the lateral shift of the transmitted beam can be negative. This is a novel phenomenon that is reversed in comparison with the geometrical optic prediction according to Snell's law of refraction. A Gaussian-shaped beam is analyzed in the paraxial approximation, and a comparison with numerical simulations is made. Finally, an explanation for the negativity of the lateral shift is suggested, in terms of the interaction of boundary effects of the slab's two interfaces with air.

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Soft-Matter Optical Spectroscopy

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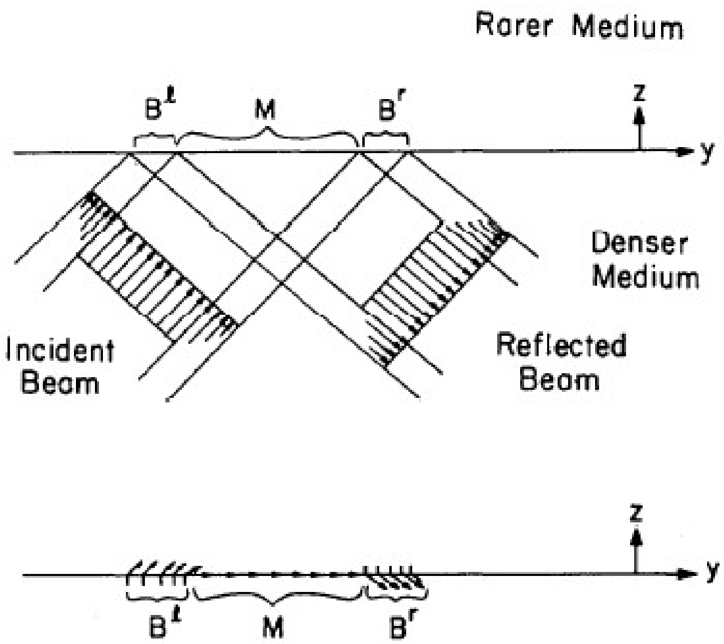
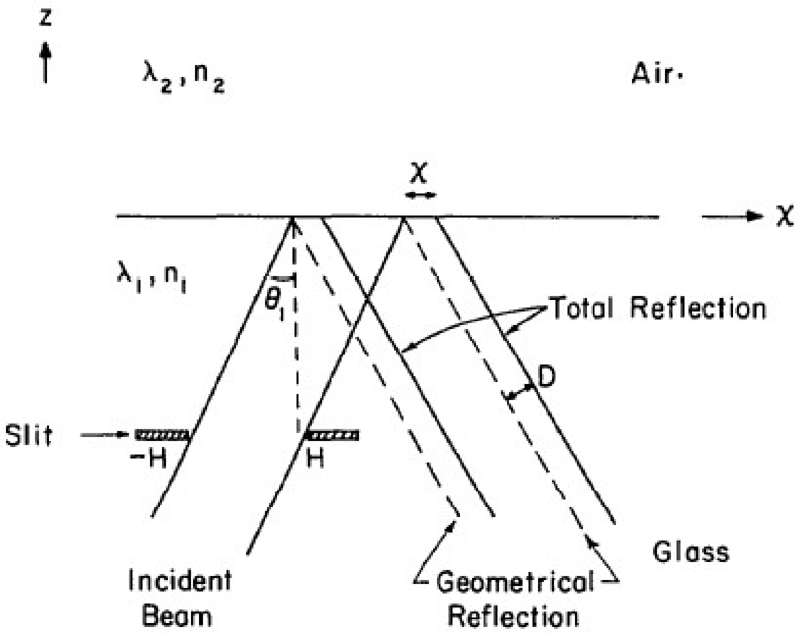
Goos-Hanchen Shift



$B^l : \theta \le \theta_c$
– the energy enters into rarer medium
$M : \theta \simeq \theta_c$
– the evanescent wave propagete
$B^r : \theta \ge \theta_c$
– the energy is given back to this region

Gives a lateral shift of the reflected beam

Goos-Hanchen Shift



Consider two rays that passing through the slit

$$\psi = \exp[i(k_x x + \phi)] + \exp[i((k_x + \Delta k_x)x + \phi + \Delta\phi)]$$
$$= \exp[i(k_x x + \phi)]\{1 + \exp i(\Delta k_x x + \Delta\phi)\}$$

ϕ is phase change by total internal reflection

The maxima case of the total intensity is,

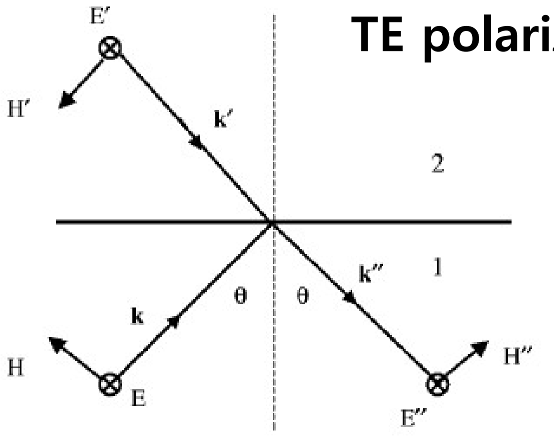
$$\Delta k_x x + \Delta\phi = 2\pi\nu$$

Compare with the perfectly silvered surface, $\Delta k_x x_{silvered} = 2\pi\nu$

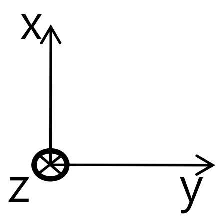
$$\mathrm{X} = x - x_{silvered} = -\frac{\Delta\phi}{\Delta k_x}, \text{infinitesimally } \mathrm{X} = -\frac{d\phi}{dk_x}$$

$$So,\ \mathrm{D} = -\cos\theta \frac{d\phi}{dk_x}$$

Goos-Hanchen Shift in LHM



TE polarization



$$k_x^2+k_y^2=k_x''^2+k_y''^2=k_1^2=\epsilon_1\mu_1\omega^2,$$

$$k_x'^2+k_y'^2=k_2^2=\epsilon_2\mu_2\omega^2.$$

$$\mu_1>0,\varepsilon_1>0$$
$$\mu_2<0,\varepsilon_2<0$$

$$k_x=k_1\cos\theta,\quad k_y=k_1\sin\theta,\quad k_1>0,$$

At the boundary, x=0 $k_y^{'}=k_y^{''}=k_y$

$$k_x^{'}=\pm\sqrt{\varepsilon_2\mu_2\omega^2-k_1^{\,2}\sin^2\theta},k_x^{''}=\pm k_1\cos\theta$$

In the region of incident angle that exceed the critical angle,

$$\varepsilon_2\mu_2\omega^2-k_1^{\,2}\sin^2\theta<0$$
$$k_x^{'}=+i\sqrt{-\varepsilon_2\mu_2\omega^2+k_1^{\,2}\sin^2\theta}$$

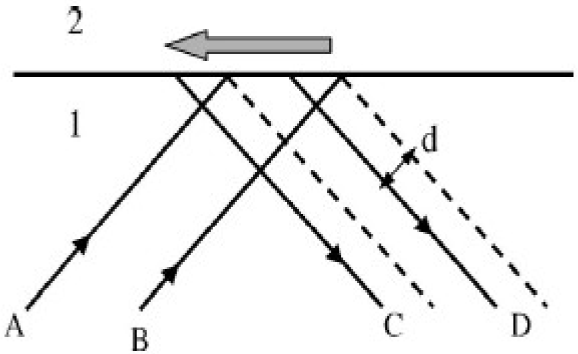
Goos-Hanchen Shift in LHM

Let's check the boundary condition of electric and magnetic field
At the interface,

$$E + E'' = E',$$
$$H - H'' = H'$$

$$\frac{k_x}{\mu_1}(E - E'') = \frac{k_x'}{\mu_2} E' = -i\frac{\sqrt{\varepsilon_1 \mu_1}}{|\mu_2|}\sqrt{\sin^2\theta - \frac{\varepsilon_2 \mu_2}{\varepsilon_1 \mu_1}}$$

$$\sqrt{R} = \frac{E''}{E} = -\frac{k_x + i\left|\frac{\mu_1}{\mu_2}\right|\sqrt{k_1^2 - k_2^2 - k_x^2}}{k_x - i\left|\frac{\mu_1}{\mu_2}\right|\sqrt{k_1^2 - k_2^2 - k_x^2}}$$



Goos-Hanchen Shift in LHM

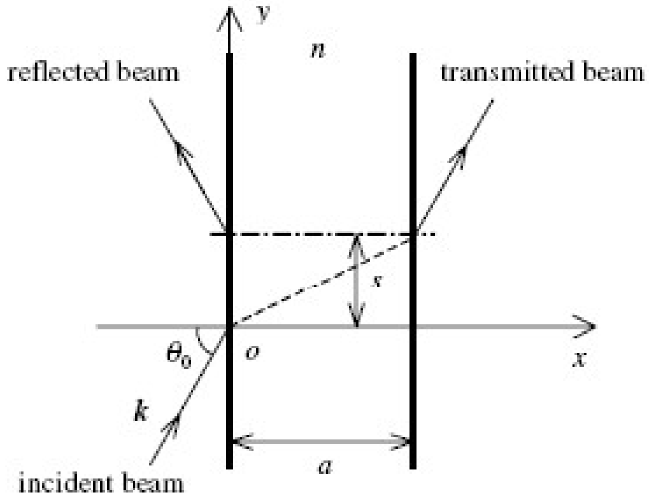
Due to the negative permeability, the phase shift of reflected beam is opposite to the RHM

Cause the Goos-Hanchen Shift in left direction

$$d = \frac{2}{k_1} \frac{|\mu_2/\mu_1| \sin\theta}{\left[(\mu_2/\mu_1)^2 \cos^2\theta + \sin^2\theta - \frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}\right] \sqrt{\sin^2\theta - \frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}}.$$

Calculated Goos-Hanchen Shift

Goos-Hanchen Shift in dielectric slab



Using the boundary condition at x=0 and x=a, get the phase of transmitted beam

$$E_1 + E_1' = E_2 + E_2',$$
$$H_1 - H_1' = H_2 - H_2'$$

First interface

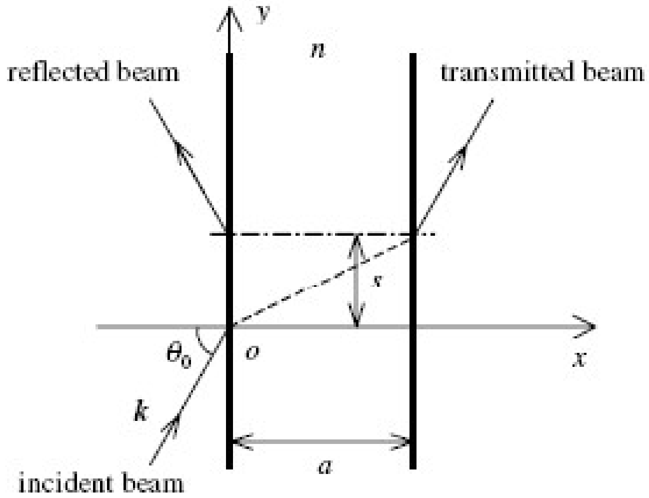
$$E_2 e^{ika} + E_2' e^{-ika} = E_T$$
$$H_1 e^{ika} - H_1' e^{-ika} = H_T$$

Second interface

$$E_1 + E_1' = (\cos ka - i\frac{n_1}{n_2}\sin ka)E_T$$
$$n_1 E_1 - n_1 E_1' = (-in_2 \sin ka + n_1 \cos ka)E_T$$

Eliminating the E_1'

Goos-Hanchen Shift in dielectric slab



$$T = \frac{E_T}{E_1} = \left[\cos k_x^{'} a - \frac{i}{2}\left(\frac{n_1}{n_2} + \frac{n_2}{n_1}\right) \sin k_x^{'} a \right]^{-1}$$

Phase contribution is,

$$\rightarrow \tan^{-1}\left(\frac{1}{2}\left(\frac{k_x}{k_x^{'}} + \frac{k_x^{'}}{k_x}\right) \tan k_x^{'} a\right)$$

The multiple reflection also should be considered,

$$\delta = 2k^{'} a \cos\theta + n\pi = 2k_x^{'} a + n\pi = 2\pi N$$

So total change of phase is,

$$\varphi(k_y) = \mathrm{int}\left(\frac{k_x^{\prime} a}{\pi} + \frac{1}{2}\right)\pi + \tan^{-1}\left[\frac{1}{2}\left(\frac{k_x^{\prime}}{k_x} + \frac{k_x}{k_x^{\prime}}\right)\tan k_x^{\prime} a\right]$$

Goos-Hanchen Shift in dielectric slab

Then, Goos-Hanchen Shift is given as,

$$S = -\frac{d\phi}{dk_y}$$

$$s = \frac{2k_{y0}a}{k_{x0}} \frac{k_{x0}^2(k_{x0}^2 + k_{x0}'^2)/k_0^4 - \sin(2k_{x0}'a)/2k_{x0}'a}{4k_{x0}^2 k_{x0}'^2/k_0^4 + \sin^2 k_{x0}'a}$$

And the sign of the numerator determines the direction of Goos-Hanchen Shift,

$$k_{x0}^2(k_{x0}^2 + k_{x0}'^2)/k_0^4 < \sin(2k_{x0}'a)/2k_{x0}'a$$ **- Left shift**

It can be converted into the necessary condition of incident angle,

$$\cos\theta_0 < \left(\frac{n^2 - 1}{2}\right)^{1/2} \equiv \cos\theta_t$$

Goos-Hanchen Shift in dielectric slab

So, left Goos-Hanchen Shift can be easily observed in dielectric media having large refractive index ,

Fixing the incident angle that satisfies the necessary condition, Goos-Hanchen Shift versus the thickness of the slab can be calculated

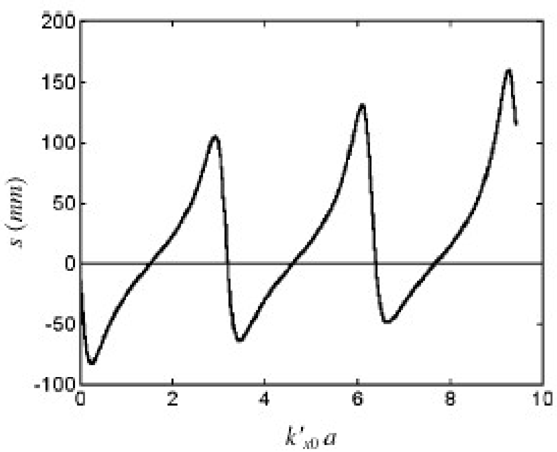


FIG. 2 Dependence of the lateral shift s on the thickness a of the slab, where the denser medium is chosen to be perspex of refractive index $n = 1.605$ at wavelength $\lambda = 32.8$ mm, the incidence angle is $\theta_0 = 80.2°$, a is rescaled by $k'_{x0}a$.

Goos-Hanchen Shift in dielectric slab

Also the phase change of the reflected wave can be derived as below,

$$R(k_y) = \frac{\exp(i\pi/2)}{4g^2}\left(\frac{k'_x}{k_x} - \frac{k_x}{k'_x}\right) \times \left[\sin 2k'_x a + i\left(\frac{k'_x}{k_x} + \frac{k_x}{k'_x}\right)\sin^2 k'_x a\right].$$

$$\sin 2k'_x a + i\left(\frac{k'_x}{k_x} + \frac{k_x}{k'_x}\right)\sin^2 k'_x a.$$

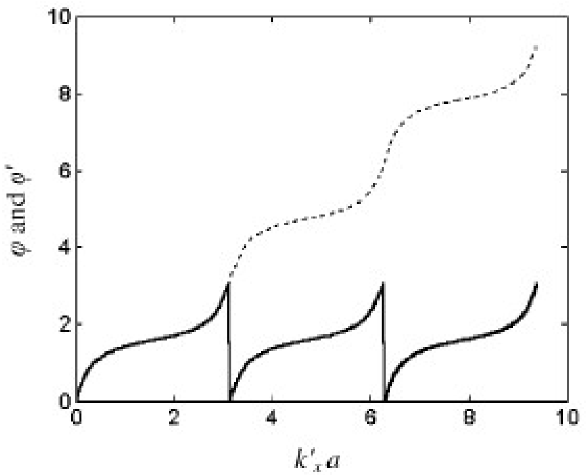


FIG. 3. Dependence of the phases φ' and φ on $k'_x a$, where $\lambda = 32.8$ mm, $n = 1.605$, $\theta = 80.2°$. φ' is shown by the real curve, and φ is shown by the dotted curve.

Phase change of reflected and transmitted beam versus thickness can be calculated,