# *Was Einstein Wrong?*

## - Measurement of the Instantaneous Velocity of a Brownian Particle

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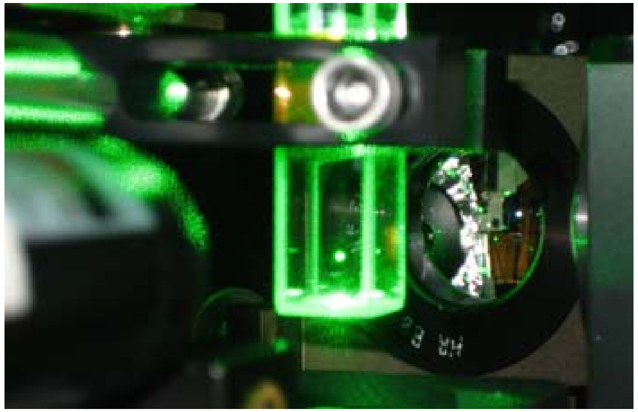


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Brownian motion of particles impacts many branches of science. We report on the Brownian motion of micronsized beads of glass held in air in an optical tweezer, over a wide range of pressures, and measure the instantaneous velocity of a Brownian particle. **Our results provide direct verification of the energy equipartition theorem for a Brownian particle.** For short times, the ballistic regime of Brownian motion is observed, in contrast to the usual diffusive regime. We discuss the applications of these methods towards cooling the center of mass motion of a bead in vacuum to the quantum ground motional state.

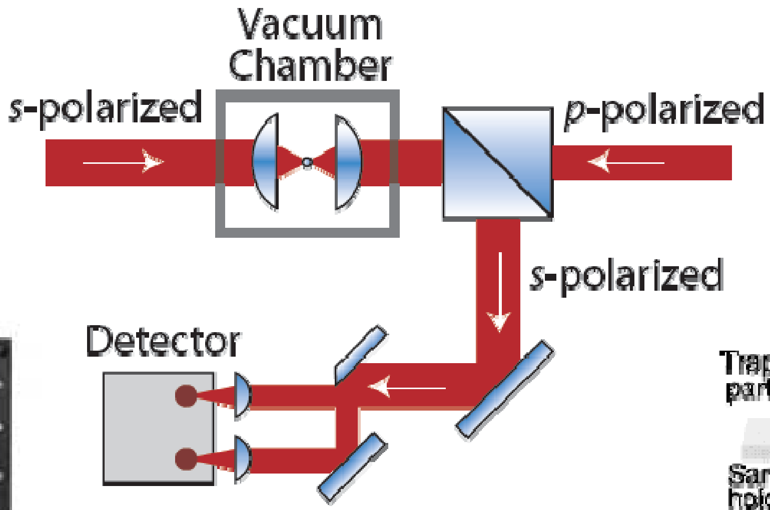
**Mark G. Raizen**

**Review : Equipartition Theorem**

$$H_{\mathrm{kin}} = \tfrac{1}{2}m|\mathbf{v}|^2 = \tfrac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right), \qquad v_{\mathrm{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}},$$

# *Setup*

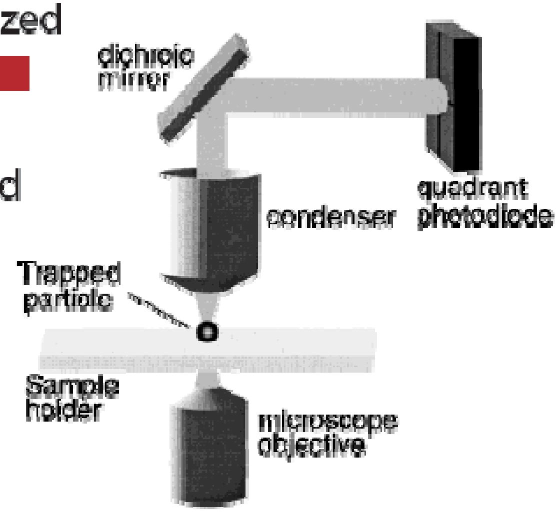
Beam Wavelength : 1064nm
NA 0.68, f = 3.1mm aspheric lens
Beam waist 2.2μm (s-pol), 3.0μm (p-pol)
160MHz (0.5pm) frequency differ




Sampling rate

Position -
2MHz (500ns)

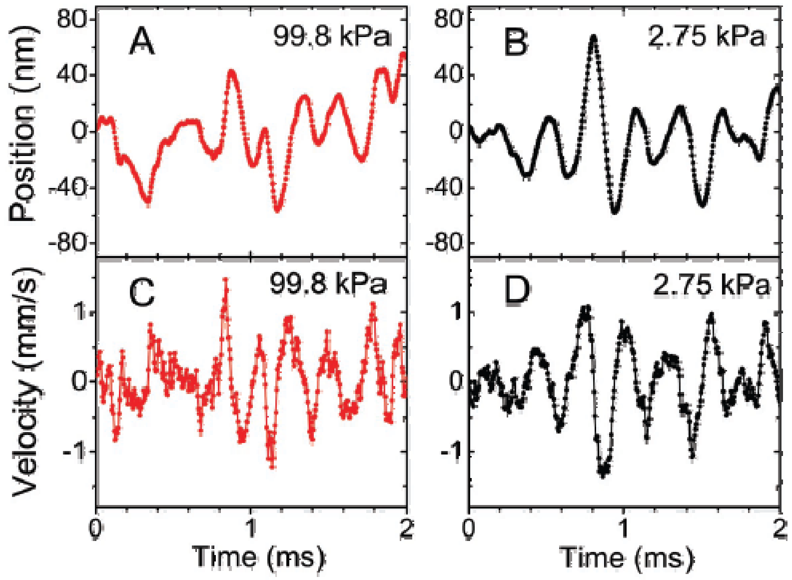
Velocity –
5μs


Photonic Force Microscope

The Position traces of the bead at two pressures appear very similar.

**cf>**
**1atm = 101.325 kPa**



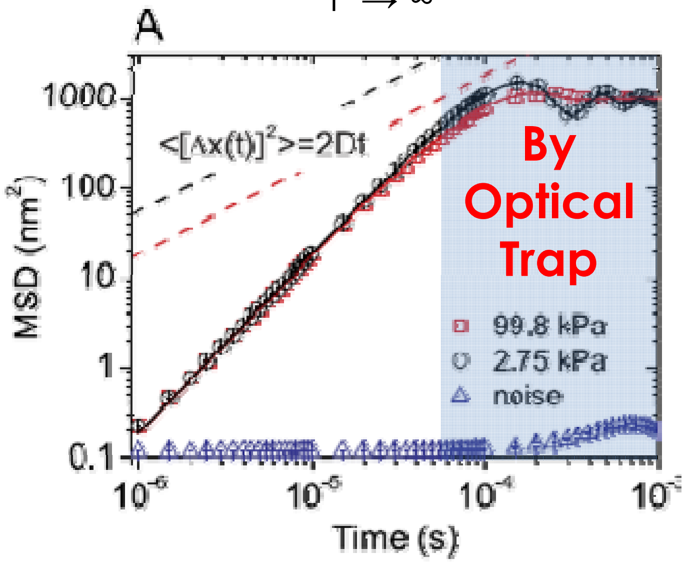

The velocity traces of the bead at two pressures appear very different.

**Momentum relaxation time is shorter at higher pressure !**

# *Mean Square Displacement*

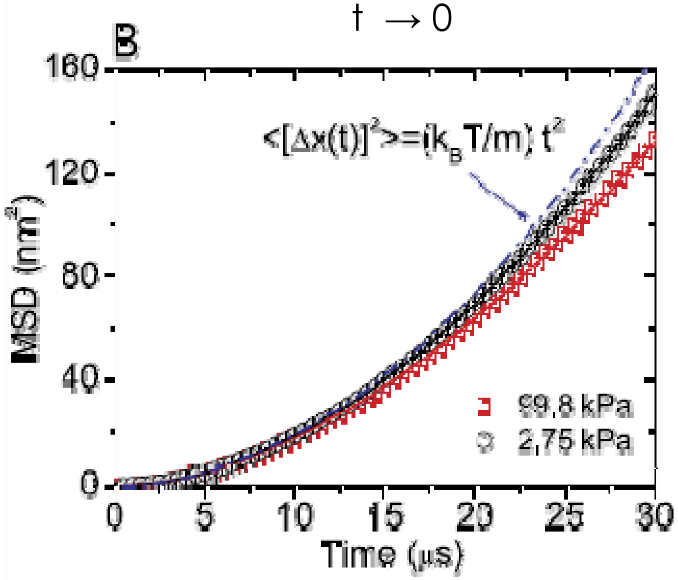
Diffusive Brownian Motion

$t \rightarrow \infty$




Ballistic (nondiffusive) Brownian Motion

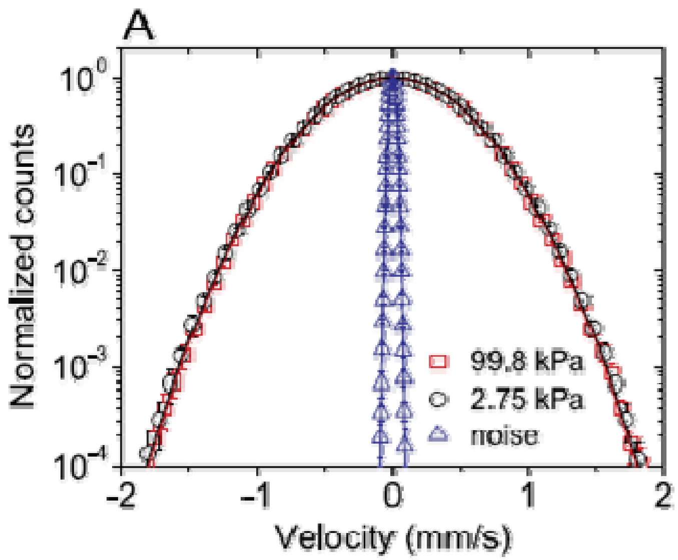
$t \rightarrow 0$




$$\left\langle [\Delta x(t)]^2 \right\rangle = \frac{2k_B T}{m\omega_0^2}[1 - e^{-t/2\tau_p}(\cos\omega_1 t + \frac{\sin\omega_1 t}{2\omega_1\tau_p})]$$

Brownian Particle in an underdamped harmonic trap
(from Langevin Equation)

# *Instantaneous Velocity Measurement*




**Solid line : Maxwell-Boltzmann Distribution**

$$v_{rms,99.8kPa} = 0.422\text{mm/s}$$

$$v_{rms,2.75kPa} = 0.425\text{mm/s}$$

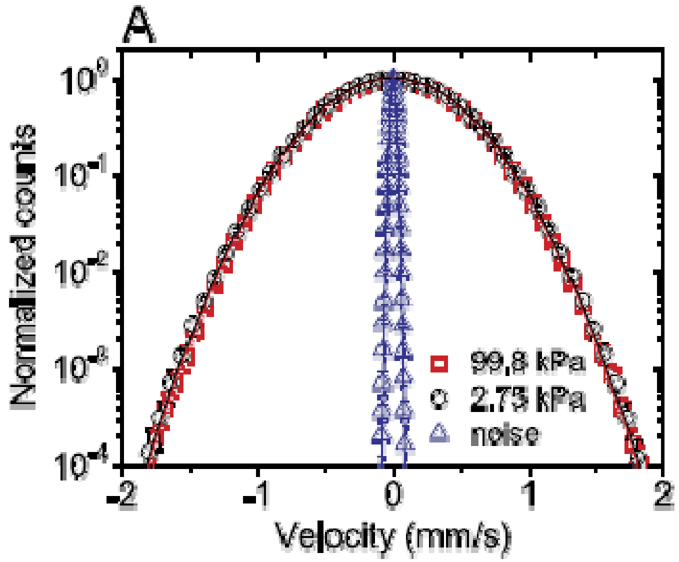
$$v_{rms,\text{equipartition theorem}} = \sqrt{(k_B T/m)} = 0.429\text{mm/s}$$

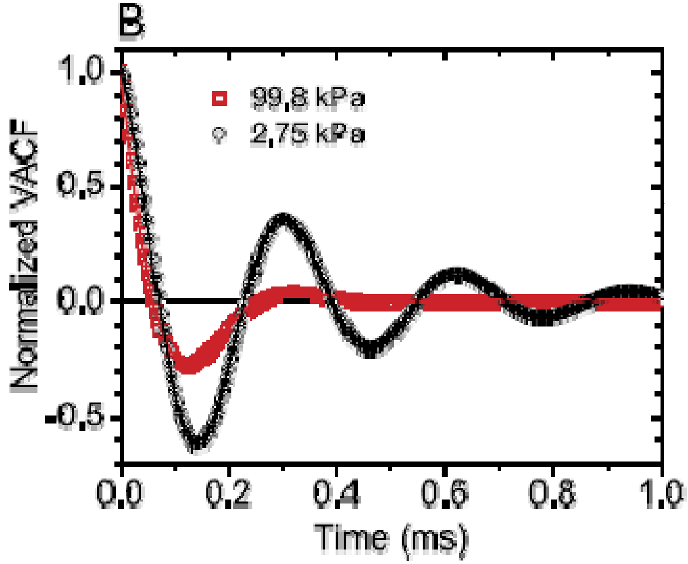
Direct evidences for

i) Maxwell-Boltzmann Distribution

ii) Equipartition Theorem

# *Instantaneous Velocity Measurement*

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$v_{rms,equipartition\ theorem} = \sqrt{(k_B T/m)}$

**= 0.429mm/s**

**Velocity AutoCorrelation Function**

$$\psi(t) = e^{-t/2\tau_p}(\cos\omega_1 t + \frac{\sin\omega_1 t}{2\omega_1\tau_p})$$

$\tau_p$ **= 48 μs at 99.8kPa**

$\tau_p$ **= 147 μs at 2.75kPa**

# *Final Goal*

*We have a longer term goal for this experiment:* ***An optically trapped bead in vacuum is an ideal system for investigating quantum effects in a mechanical system****, due to its near-perfect isolation from the thermal environment . Combining feedback cooling and cavity cooling, we expect to cool the motion of a bead starting from room temperature to the quantum regime where we can study the creation and decoherence of Schrödinger cat states.*

http://george.ph.utexas.edu/research.html#microspheres