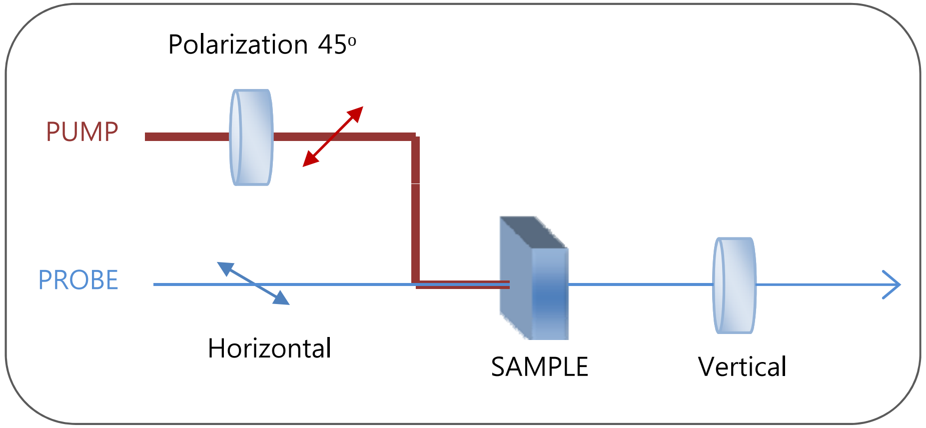
Balanced Detection

2009 9 4 junheesun

Homodyne

C



incident beam vector : horizontal $\begin{pmatrix}1\\0\end{pmatrix}$

sample (relative phase changer) :

$$\begin{pmatrix}\cos 45^\circ & -\sin 45^\circ\\ \sin 45^\circ & \cos 45^\circ\end{pmatrix}\begin{pmatrix}e^{i\phi_x} & 0\\ 0 & e^{i\phi_y}\end{pmatrix}\begin{pmatrix}\cos 45^\circ & \sin 45^\circ\\ -\sin 45^\circ & \cos 45^\circ\end{pmatrix}$$

$$=\begin{pmatrix}\frac{e^{i\phi_x}}{2}+\frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2}-\frac{e^{i\phi_y}}{2}\\ \frac{e^{i\phi_x}}{2}-\frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2}+\frac{e^{i\phi_y}}{2}\end{pmatrix}$$

analyzer : vertical $\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$

$$\begin{pmatrix}E_h\\ E_v\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\frac{e^{i\phi_x}}{2}+\frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2}-\frac{e^{i\phi_y}}{2}\\ \frac{e^{i\phi_x}}{2}-\frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2}+\frac{e^{i\phi_y}}{2}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\ \frac{e^{i\phi_x}}{2}-\frac{e^{i\phi_y}}{2}\end{pmatrix}$$

$$I \propto (E_hE_h^{\ *})+(E_vE_v^{\ *})$$

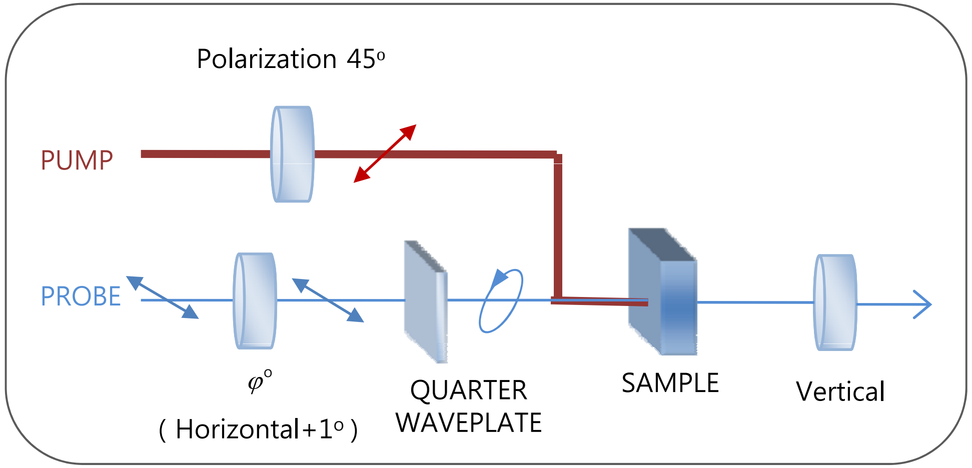
$$E_vE_v^{\ *}=(\frac{e^{i\phi_x}}{2}-\frac{e^{i\phi_y}}{2})\times(\frac{e^{-i\phi_x}}{2}-\frac{e^{-i\phi_y}}{2})$$

$$=\frac{1}{4}\cos\phi_x^{\ 2}-\frac{1}{2}\cos\phi_x\cos\phi_y+\frac{1}{4}\cos\phi_y^{\ 2}+\frac{1}{4}\sin\phi_x^{\ 2}-\frac{1}{2}\sin\phi_x\sin\phi_y+\frac{1}{4}\sin\phi_y^{\ 2}$$

$$=\sin(\frac{\phi_x-\phi_y}{2})2$$

$$I \propto \sin(\frac{\phi_x-\phi_y}{2})^2$$

Heterodyne



incident beam vector : horizontal $\begin{pmatrix}1\\0\end{pmatrix}$

polarizer : angle φ° $(46 \le \varphi)$

$$\begin{pmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\cos\varphi & \sin\varphi\\ -\sin\varphi & \cos\varphi\end{pmatrix}$$

$$= \begin{pmatrix}\cos^2\varphi & \sin\varphi\cos\varphi\\ \sin\varphi\cos\varphi & \sin^2\varphi\end{pmatrix}$$

quarter waveplate : fast axis $\rightarrow$ horizontal $\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$

sample (relative phase changer) :

$$\begin{pmatrix}\cos 45^{\circ} & -\sin 45^{\circ}\\ \sin 45^{\circ} & \cos 45^{\circ}\end{pmatrix}\begin{pmatrix}e^{i\phi_x} & 0\\ 0 & e^{i\phi_y}\end{pmatrix}\begin{pmatrix}\cos 45^{\circ} & \sin 45^{\circ}\\ -\sin 45^{\circ} & \cos 45^{\circ}\end{pmatrix}$$

$$= \begin{pmatrix}\frac{e^{i\phi_x}}{2}+\frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2}-\frac{e^{i\phi_y}}{2}\\ \frac{e^{i\phi_x}}{2}-\frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2}+\frac{e^{i\phi_y}}{2}\end{pmatrix}$$

analyzer : vertical $\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$

$$\begin{pmatrix} E_h \\ E_v \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{e^{i\phi_x}}{2} + \frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2} - \frac{e^{i\phi_y}}{2} \\ \frac{e^{i\phi_x}}{2} - \frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2} + \frac{e^{i\phi_y}}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} \cos^2\varphi & \sin\varphi\cos\varphi \\ \sin\varphi\cos\varphi & \sin^2\varphi \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \cos\varphi\left((\frac{e^{i\phi_x}}{2} - \frac{e^{i\phi_y}}{2})\cos\varphi + i(\frac{e^{i\phi_x}}{2} + \frac{e^{i\phi_y}}{2})\sin\varphi \right) \end{pmatrix}$$

$$\therefore I \propto (E_h E_h^{*}) + (E_v E_v^{*})$$

$$E_v E_v^{*} = \left(\cos\varphi\left((\frac{e^{i\phi_x}}{2} - \frac{e^{i\phi_y}}{2})\cos\varphi + i(\frac{e^{i\phi_x}}{2} + \frac{e^{i\phi_y}}{2})\sin\varphi \right)\right) \times \left(\cos\varphi\left((\frac{e^{-i\phi_x}}{2} - \frac{e^{-i\phi_y}}{2})\cos\varphi - i(\frac{e^{-i\phi_x}}{2} + \frac{e^{-i\phi_y}}{2})\sin\varphi \right)\right)$$

$$= \frac{1}{4}\cos\varphi^4\cos\phi_x^{\ 2} - \frac{1}{2}\cos\varphi^4\cos\phi_x\cos\phi_y + \frac{1}{4}\cos\varphi^4\cos\phi_y^{\ 2} + \frac{1}{4}\cos\varphi^2\cos\phi_x^{\ 2}\sin\varphi^2$$

$$+ \frac{1}{2}\cos\varphi^2\cos\phi_x\cos\phi_y\sin\varphi^2 + \frac{1}{4}\cos\varphi^2\cos\phi_y^{\ 2}\sin\varphi^2 + \cos\varphi^3\cos\phi_y\sin\varphi\sin\phi_x$$

$$+ \frac{1}{4}\cos\varphi^4\sin\phi_x^{\ 2} + \frac{1}{4}\cos\varphi^2\sin\varphi^2\sin\phi_x^{\ 2} - \cos\varphi^3\cos\phi_x\sin\varphi\sin\phi_y - \frac{1}{2}\cos\varphi^4\sin\phi_x\sin\phi_y$$

$$+ \frac{1}{2}\cos\varphi^2\sin\varphi^2\sin\phi_x\sin\phi_y + \frac{1}{4}\cos\varphi^4\sin\phi_y + \frac{1}{4}\cos\varphi^2\sin\varphi^2\sin\phi_y$$

$$= 0.000306 - 0.000153\sin(\frac{\phi_x - \phi_y}{2})^2 + (5.35\times10^{-6})\sin(\phi_x - \phi_y)$$

$$I \propto 0.499752 - 0.499448\cos(\phi_x - \phi_y) + 0.0174395\sin(\phi_x - \phi_y)$$

$$\propto \cos(\varphi)^2\sin[\frac{1}{2}(2\varphi + \phi_x - \phi_y)]^2$$

(※ $\cos\varphi$, $\sin\varphi$ 의 Talyor expansion

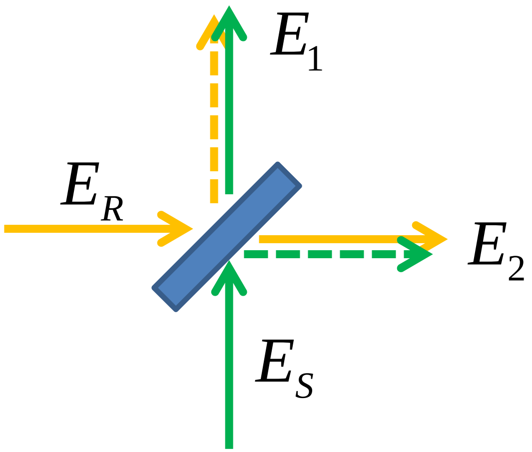
$\varphi = \delta$ (※ $\delta \approx 1^o \approx 0.0175$rad)

$$\cos(\delta) = \cos 0 - \delta\sin 0 - \frac{1}{2}\delta^2\cos 0 + \cdots \cong 1 - \frac{1}{2}\delta^2 = 0.9998$$

$$\sin(\delta) = \sin 0 + \delta\cos 0 - \frac{1}{2}\delta^2\sin 0 + \cdots \cong \delta = 0.0175$$)

Balanced detection

Homodyne interferometer



Signal beam $E_S = E_{SO} e^{-i(wt - \phi_S + \phi_m)}$

Reference beam $E_R = E_{RO} e^{-i(wt - \phi_R)}$

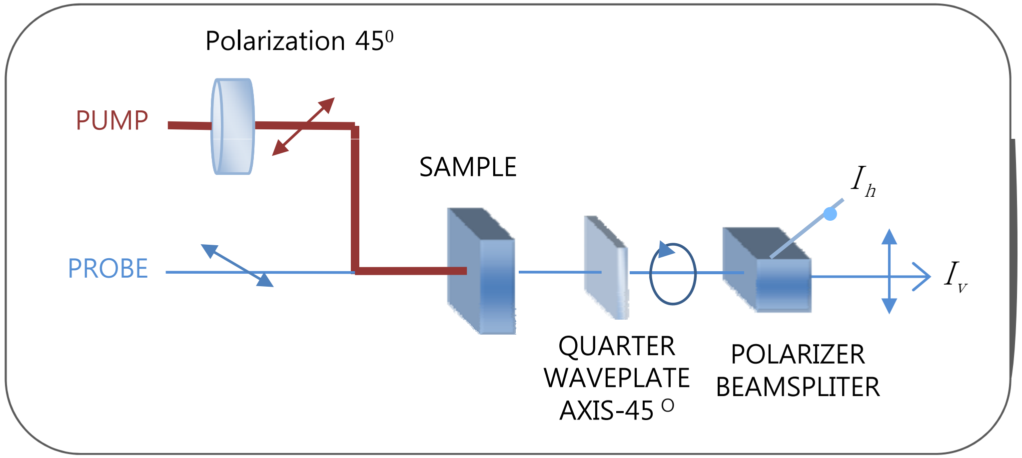
$$I_1 = I_{SO} + I_{RO} + \sqrt{I_{SO} I_{RO}} \sin(\Delta\phi_0 - \phi_m)$$

$$I_2 = I_{SO} + I_{RO} - \sqrt{I_{SO} I_{RO}} \sin(\Delta\phi_0 - \phi_m)$$

$$I_{12} = 2\sqrt{I_{SO} I_{RO}} \sin(\Delta\phi_0 - \phi_m)$$: differential amplifier

$$\Delta\phi_0 = \phi_S - \phi_R = 2n\pi \quad \text{(maximum)}$$

Balanced detection



incident beam vector : horizontal $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

sample (relative phase changer) :

$$\begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} \begin{pmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{pmatrix} \begin{pmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{pmatrix}$$

$$= \begin{pmatrix} \frac{e^{i\phi_x}}{2} + \frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2} - \frac{e^{i\phi_y}}{2} \\ \frac{e^{i\phi_x}}{2} - \frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2} + \frac{e^{i\phi_y}}{2} \end{pmatrix}$$

quarter waveplate : fast axis $\rightarrow -45^\circ$ $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$

$$\begin{pmatrix} E_h \\ E_v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \begin{pmatrix} \frac{e^{i\phi_x}}{2} + \frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2} - \frac{e^{i\phi_y}}{2} \\ \frac{e^{i\phi_x}}{2} - \frac{e^{i\phi_y}}{2} & \frac{e^{i\phi_x}}{2} + \frac{e^{i\phi_y}}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{(\frac{1}{2} - \frac{i}{2})e^{i\phi_x}}{\sqrt{2}} + \frac{(\frac{1}{2} + \frac{i}{2})e^{i\phi_y}}{\sqrt{2}} \\ \frac{(\frac{1}{2} - \frac{i}{2})e^{i\phi_x}}{\sqrt{2}} - \frac{(\frac{1}{2} + \frac{i}{2})e^{i\phi_y}}{\sqrt{2}} \end{pmatrix}$$

$$I_h - I_v \propto (E_h E_h^*) - (E_v E_v^*)$$

$$(E_h E_h^*) - (E_v E_v^*) = \left(\frac{(\frac{1}{2} - \frac{i}{2})e^{i\phi_x}}{\sqrt{2}} + \frac{(\frac{1}{2} + \frac{i}{2})e^{i\phi_y}}{\sqrt{2}} \right) \times \left(\frac{(\frac{1}{2} - \frac{-i}{2})e^{-i\phi_x}}{\sqrt{2}} + \frac{(\frac{1}{2} + \frac{-i}{2})e^{-i\phi_y}}{\sqrt{2}} \right)$$

$$- \left(\frac{(\frac{1}{2} - \frac{i}{2})e^{i\phi_x}}{\sqrt{2}} - \frac{(\frac{1}{2} + \frac{i}{2})e^{i\phi_y}}{\sqrt{2}} \right) \times \left(\frac{(\frac{1}{2} - \frac{-i}{2})e^{-i\phi_x}}{\sqrt{2}} - \frac{(\frac{1}{2} + \frac{-i}{2})e^{-i\phi_y}}{\sqrt{2}} \right)$$

$$= \cos\phi_y \sin\phi_x - \cos\phi_x \sin\phi_y$$

$$= \sin(\phi_x - \phi_y)$$

$$I_h - I_v \propto \sin(\phi_x - \phi_y)$$

MODEL SR850
DSP Lock-In Amplifier

Stanford Research Systems

output

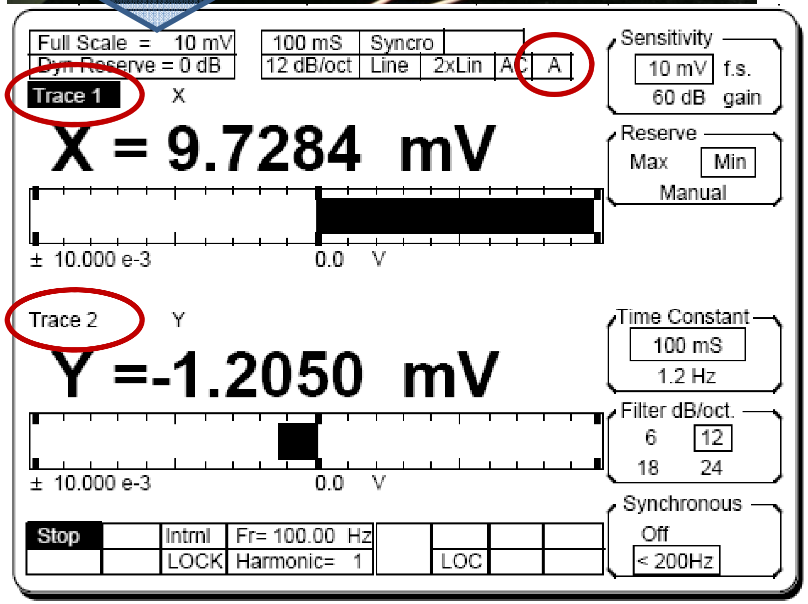
Channel 1 Output	X, R, θ, or Trace 1-4. Traces are defined as A•B/C or A•B/C2 where A, B, and C are selected from the quantities Unity, X, Y, R, θ, Xnoise, Ynoise, Rnoise, Aux Inputs 1 through 4, or Frequency. Output Voltage: ±10 V full scale. 10 mA max output current.
Channel 2 Output	Y, R, θ, or Trace 1-4. Traces are defined as A•B/C or A•B/C2 where A, B, and C are selected from the quantities Unity, X, Y, R, θ, Xnoise, Ynoise, Rnoise, Aux Inputs 1 through 4, or Frequency. Output Voltage: ±10 V full scale. 10 mA max output current.

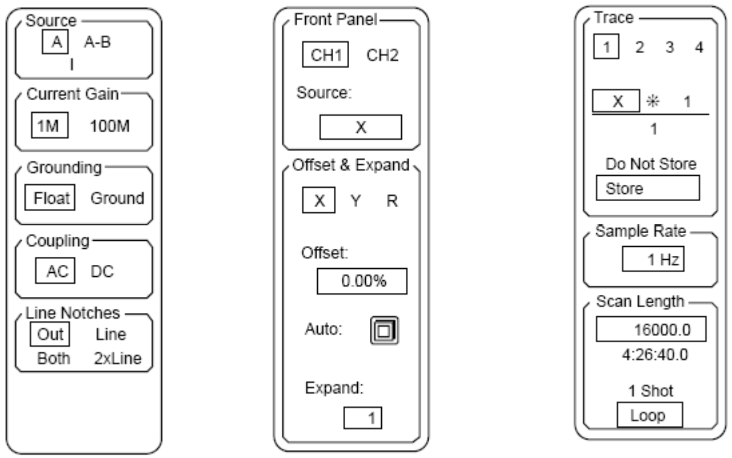
Trace 1	X
Trace 2	Y
Trace 3	R
Trace 4	θ

input

SIGNAL CHANNEL

Voltage Inputs — Single-ended (A) or differential (A-B).





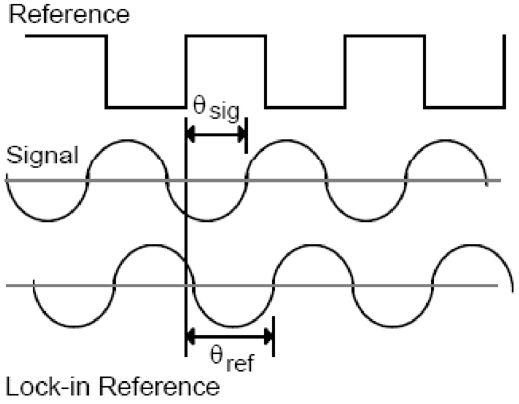
Source

CH1 can be proportional to X, R, θ, Trace1, Trace2, Trace3, or Trace 4.
CH2 can be proportional to Y, R, θ, Trace1, Trace2, Trace3, or Trace 4.

When the unit is reset, the traces are defined as
Trace 1 = X, Trace 2 = Y, Trace 3 = R, Trace 4 = θ.

A•B/C or A•B/C2

The SR850 generates its own sine wave, shown as the lock-in reference below. The lock-in reference is $V_L \sin(\omega_L t + \theta_{ref})$.



The SR850 amplifies the signal and then multiplies it by the lock-in reference using a phase-sensitive detector or multiplier. The output of the PSD is simply the product of two sine waves.

$$V_{psd} = V_{sig} V_L \sin(\omega_r t + \theta_{sig}) \sin(\omega_L t + \theta_{ref})$$

$$= 1/2\, V_{sig} V_L \cos([\omega_r - \omega_L]t + \theta_{sig} - \theta_{ref}) - 1/2\, V_{sig} V_L \cos([\omega_r + \omega_L]t + \theta_{sig} + \theta_{ref})$$

The PSD output is two AC signals, one at the difference frequency ($\omega_r - \omega_L$) and the other at the sum frequency ($\omega_r + \omega_L$).

If the PSD output is passed through a low pass filter, the AC signals are removed. What will be left? In the general case, nothing. However, if ω_r equals ω_L, the difference frequency component will be a DC signal. In this case, the filtered PSD output will be

$$V_{psd} = 1/2\, V_{sig} V_L \cos(\theta_{sig} - \theta_{ref})$$

Magnitude and phase

Remember that the PSD output is proportional to $V_{sig}\cos\theta$ where $\theta = (\theta_{sig} - \theta_{ref})$. θ is the phase difference between the signal and the lock-in reference oscillator. By adjusting θ_{ref} we can make θ equal to zero, in which case we can measure V_{sig} ($\cos\theta=1$). Conversely, if θ is 90°, there will be no output at all. A lock-in with a single PSD is called a single-phase lock-in and its output is $V_{sig}\cos\theta$.

This phase dependency can be eliminated by adding a second PSD. If the second PSD multiplies the signal with the reference oscillator shifted by 90°, i.e. $V_L \sin(\omega_L t + \theta_{ref} + 90°)$, its low pass filtered output will be

$$V_{psd2} = 1/2\, V_{sig} V_L \sin(\theta_{sig} - \theta_{ref})$$

$$V_{psd2} \sim V_{sig} \sin\theta$$

Now we have two outputs, one proportional to $\cos\theta$ and the other proportional to $\sin\theta$. If we call the first output X and the second Y,

$$X = V_{sig}\cos\theta \qquad Y = V_{sig}\sin\theta$$

these two quantities represent the signal as a vector relative to the lock-in reference oscillator. X is called the 'in-phase' component and Y the 'quadrature' component. This is because when $\theta=0$, X measures the signal while Y is zero.

By computing the magnitude (R) of the signal vector, the phase dependency is removed.

$$R = (X^2 + Y^2)^{1/2} = V_{sig}$$

R measures the signal amplitude and does not depend upon the phase between the signal and lock-in reference.

A dual-phase lock-in, such as the SR850, has two PSD's, with reference oscillators 90° apart, and can measure X, Y and R directly. In addition, the phase θ between the signal and lock-in reference, can be measured according to

$$\theta = \tan^{-1}(Y/X)$$