Study of Microwave theory

- D. M. Pozar

presentation : Yoonnam Jeon

28th Feb. 2009

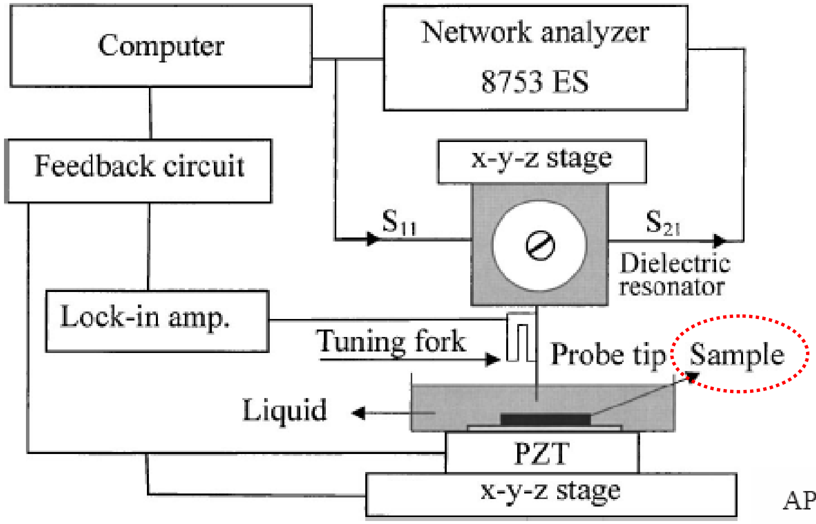
Soft-Matter Optical Spectroscopy

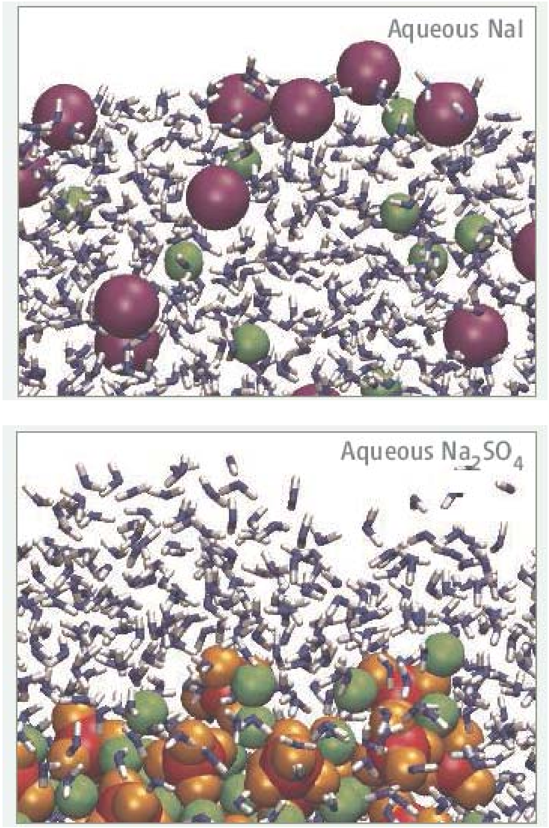
Introduction

- Is there surface enhancement of ionic salts in aqueous solution ?

yes

- Can we measure ?





APPLIED PHYSICS LETTERS **86**, 153506 (2005)

Soft-Matter Optical Spectroscopy

Introduction

APPLIED PHYSICS LETTERS **89**, 183504 (2006)

S_{11}

↓

Z^R

↓

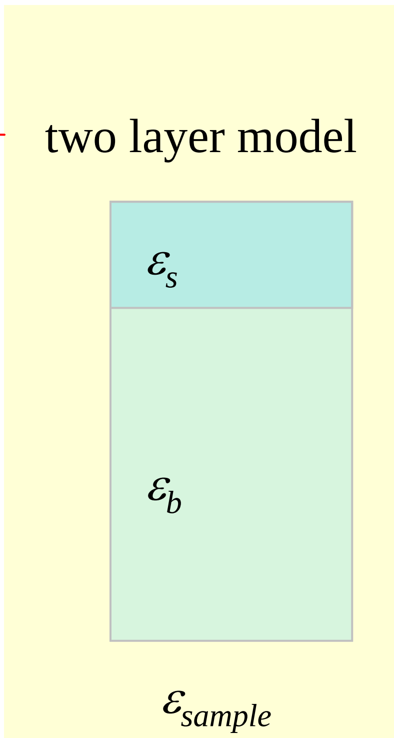
ε_{sample}

$$S_{11} = 20 \log \left| \frac{Z^R - Z_0}{Z^R + Z_0} \right|$$

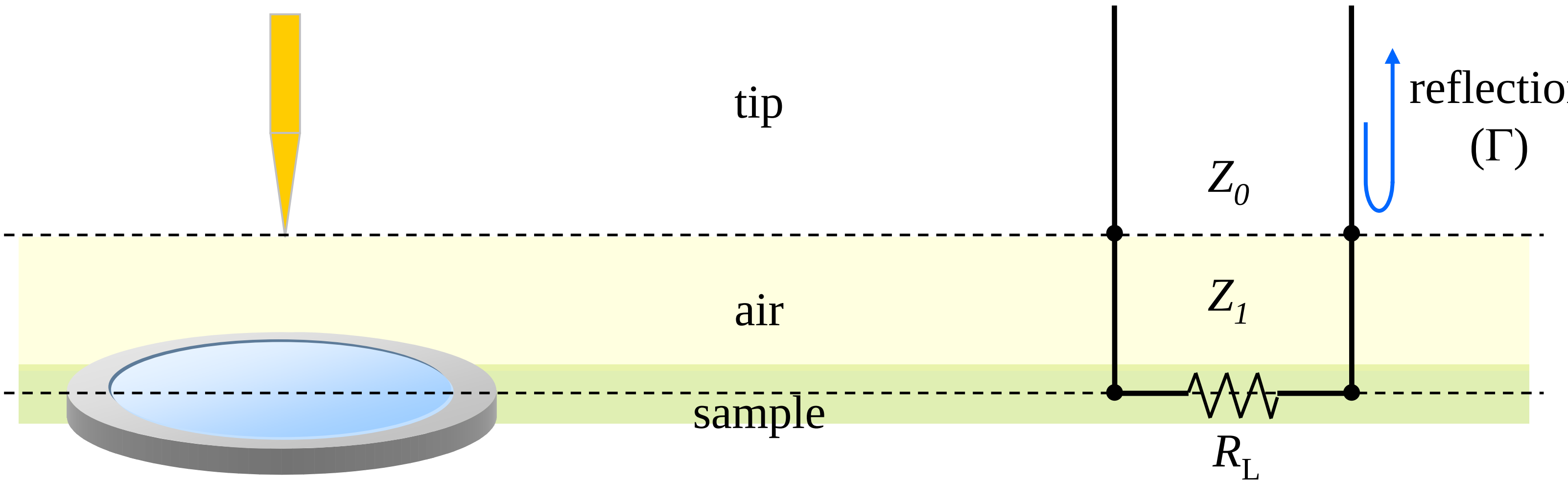
↓ ?

$$Z^R = \frac{Z_a}{\sqrt{\varepsilon_s}} \frac{1 + [\tan(k_a(v/s)\sqrt{\varepsilon_0' + c\gamma'})]^2}{1 + [(\varepsilon_0' + c\gamma')/\varepsilon_s][\tan(k_a(v/s)\sqrt{\varepsilon_0' + c\gamma'})]^2}$$

- Z_0 : effective impedance of the probe tip
- Z^R : real part of the complex impedance of solution
- k_a : wavenumber in air
- v : volume of solution
- s : surface area of solution



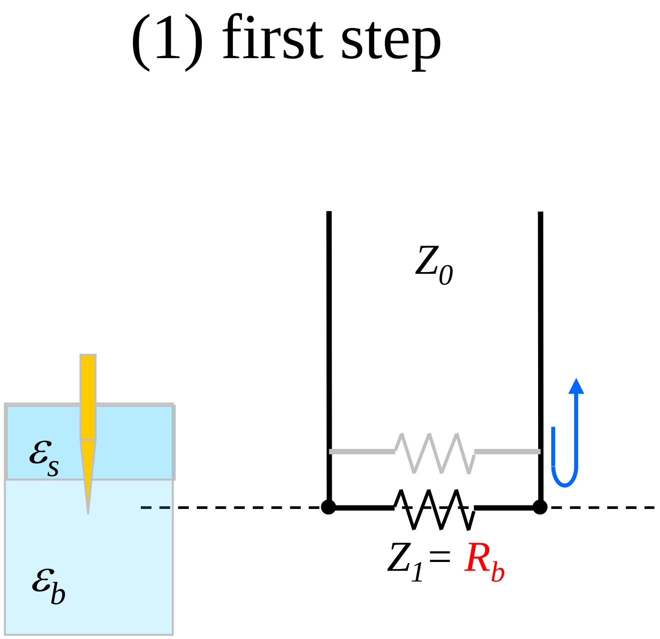
Our goal - matching with the circuit

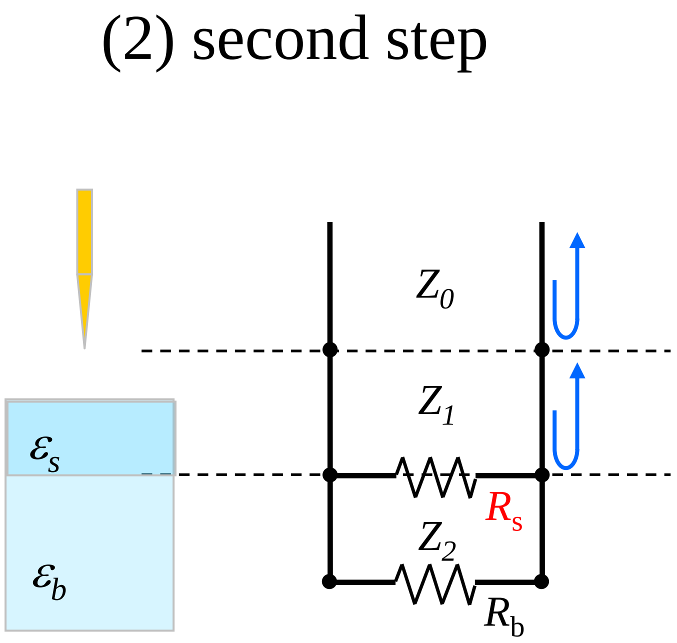


$$Z_1 = \sqrt{Z_0 R_L}$$

$$|\Gamma| = \left| \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right|$$

Outline





Wave equation - in lossless medium

$$\begin{cases} \vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H} \\ \vec{\nabla} \times \vec{H} = j\omega\varepsilon\vec{E} \end{cases} \Rightarrow \begin{cases} \vec{\nabla}^2\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0 \\ \vec{\nabla}^2\vec{H} + \omega^2\mu\varepsilon\vec{H} = 0 \end{cases}$$

$= k^2$, where $k = \omega\sqrt{\mu\varepsilon}$

sol. $\Rightarrow$

$$E_x(z) = E^+ e^{-jkz} + E^- e^{jkz}$$

$$or \quad E_x(z) = E^+ \cos(\omega t - kz) + E^- \cos(\omega t + kz)$$

$$H_y(z) = \frac{1}{\eta}(E^+ e^{-jkz} - E^- e^{jkz})$$

where $\eta = \mu\omega / k = \sqrt{\mu / \varepsilon}$; wave impedance for the plane wave

in free space, $\eta_0 = \sqrt{\mu_0 / \varepsilon_0} = 377\,\Omega$

Soft-Matter Optical Spectroscopy

Wave equation - in general lossy medium

(conductive medium)

$$\begin{cases} \vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H} \\ \vec{\nabla} \times \vec{H} = j\omega\varepsilon\vec{E} + \sigma\vec{E} \end{cases} \Rightarrow \vec{\nabla}^2\vec{E} + \omega^2\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\vec{E} = 0$$

$= -\gamma^2$, where

$$\gamma \equiv \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$$

sol.

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z}$$

$$e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z}$$

$$\rightarrow e^{-\alpha z}\cos(\omega t - \beta z)$$

decay is given by the attenuation constant, α

* phase velocity $v_p = \omega / \beta$, a wavelength $\lambda = 2\pi / \beta$
* if lossless : $\alpha = 0$, $\beta = k$, $\gamma = jk$

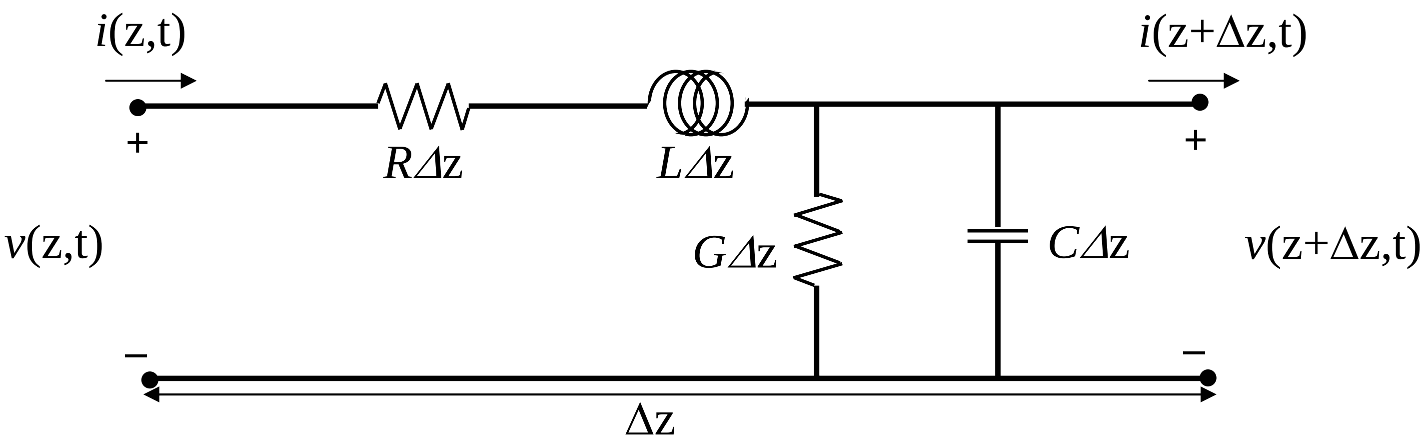
Soft-Matter Optical Spectroscopy

Summary of wave propagation - in various media

	type of Medium		
	lossless ($\varepsilon'' = \sigma = 0$)	general lossy	good conductor ($\varepsilon'' >> \varepsilon'$ or $\sigma >> \omega\varepsilon'$)
complex propagation constant	$\gamma = j\omega\sqrt{\mu\varepsilon}$	$\gamma = j\omega\sqrt{\mu\varepsilon}$ $= j\omega\sqrt{\mu\varepsilon'}\sqrt{(1-j\sigma)/\omega\varepsilon'}$	$\gamma = (1+j)\sqrt{\omega\mu\sigma/2}$
phase constant (wavenumber)	$\beta = k = \omega\sqrt{\mu\varepsilon}$	$\beta = \mathrm{Im}(\gamma)$	$\beta = \mathrm{Im}(\gamma) = \sqrt{\omega\mu\sigma/2}$
attenuation constant	$\alpha = 0$	$\alpha = \mathrm{Re}(\gamma)$	$\alpha = \mathrm{Re}(\gamma) = \sqrt{\omega\mu\sigma/2}$
impedance	$\eta = \mu\omega/k = \sqrt{\mu/\varepsilon}$	$\eta = j\mu\omega/\gamma$	$\eta = (1+j)\sqrt{\mu\omega/2\sigma}$
skin depth	$\delta_s = \infty$	$\delta_s = 1/\alpha$	$\delta_s = \sqrt{2/\mu\omega\sigma}$
wavelength	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$
phase velocity	$v_p = \omega/\beta$	$v_p = \omega/\beta$	$v_p = \omega/\beta$

* $\varepsilon = \varepsilon' - j\,\varepsilon''$

Transmission Line Theory



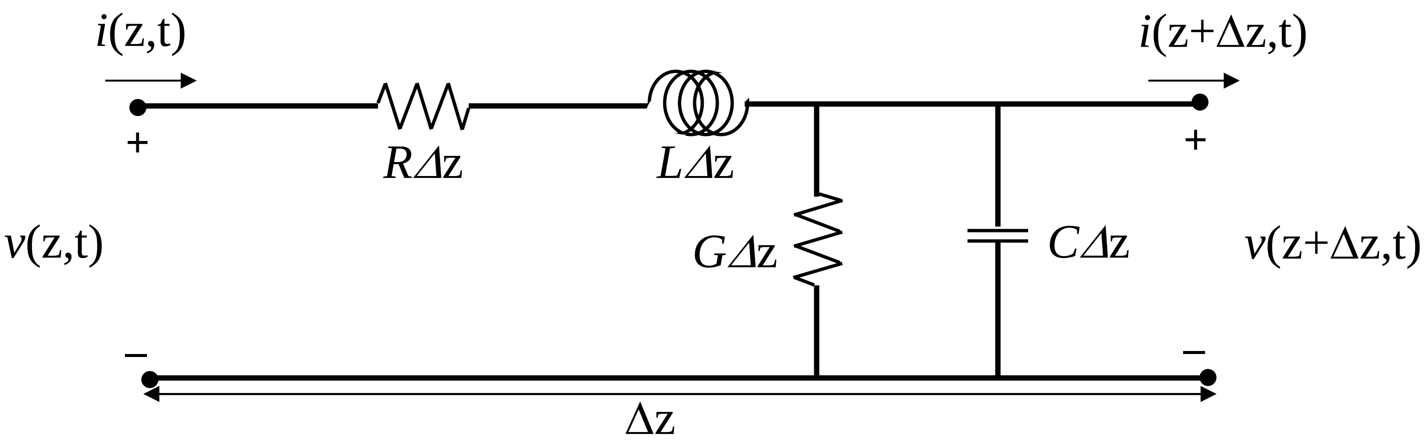
R = series resistance per unit length length, for both conductors, in Ω/m.
L = series inductance per unit length length, for both conductors, in H/m.
G = shunt conductance per unit length length, in S/m. (*G = 1/R)
C = series inductance per unit length length, in F/m.

by the Kirchhoff's law

$$v(z,t) - R\Delta z \cdot i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

$$i(z,t) - G\Delta z \cdot v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Transmission Line Theory



$$\begin{cases} v(z,t) - R\Delta z \cdot i(z,t) - L\Delta z \dfrac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0 \\ i(z,t) - G\Delta z \cdot v(z+\Delta z,t) - C\Delta z \dfrac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0 \end{cases}$$

taking the limit as $\Delta z \rightarrow 0$ gives the following

$$\begin{cases} \dfrac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\dfrac{\partial i(z,t)}{\partial t} \\ \dfrac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\dfrac{\partial v(z,t)}{\partial t} \end{cases}$$

Soft-Matter Optical Spectroscopy

Transmission Line Theory

$$\begin{cases} \dfrac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\dfrac{\partial i(z,t)}{\partial t} \\[2ex] \dfrac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\dfrac{\partial v(z,t)}{\partial t} \end{cases}$$

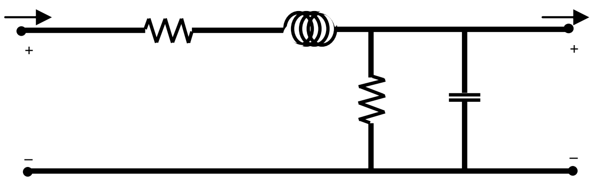
$i(z,t) = I(z)e^{jwt}$, $v(z,t) = V(z)e^{jwt}$ ⇒

$$\begin{cases} \dfrac{dV(z)}{dz} = -(R + j\omega L)I(z) \\[2ex] \dfrac{dI(z)}{dz} = -(G + j\omega C)V(z) \end{cases}$$

⇒

$$\begin{cases} \dfrac{d^2V(z)}{dz^2} = (R + j\omega L)(G + j\omega C)V(z) \\[2ex] \dfrac{d^2I(z)}{dz^2} = (R + j\omega L)(G + j\omega C)I(z) \end{cases}$$

$(R + j\omega L)(G + j\omega C) \equiv \gamma$



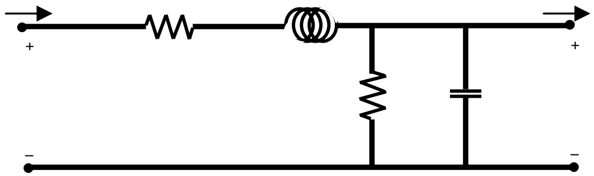
⇒

$$\begin{cases} \dfrac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \\[2ex] \dfrac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \end{cases}$$

where, $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

Transmission Line Theory

$$\begin{cases} \frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \\ \frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \end{cases}$$



sol.

$$\begin{cases} V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \end{cases}$$

z direction -z direction

$$\frac{dV(z)}{dz} = -(R + j\omega L) I(z)$$

$$\frac{dV(z)}{dz} = \frac{d}{dz}\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = -\gamma\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right)$$

$$= -(R + j\omega L) I(z)$$

$$\therefore \quad I(z) = \frac{\gamma}{R + j\omega L}\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right)$$

$\equiv 1/Z_0$

z_0 : characteristic impedance

$$= \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Soft-Matter Optical Spectroscopy

Transmission Line Theory

$$I(z) = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right) = \frac{V_0^+ e^{-\gamma z}}{Z_0} - \frac{V_0^- e^{\gamma z}}{Z_0}$$

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z}$$

$$\rightarrow e^{-\alpha z}\cos(\omega t - \beta z)$$

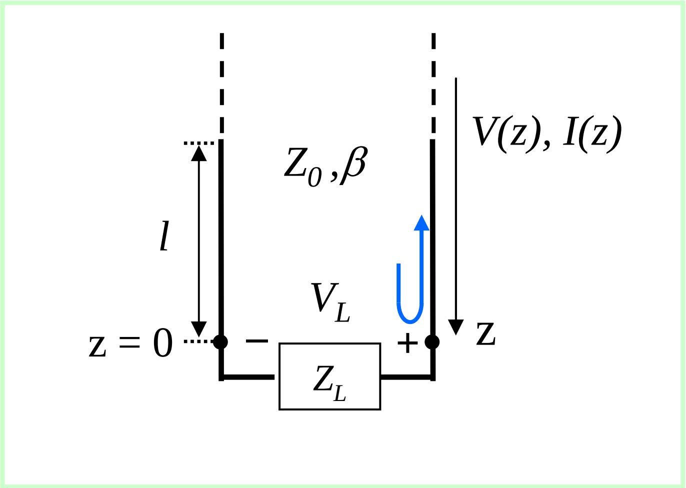
converting back to the time domain,

$$v(z,t) = \left|V_0^+\right| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + \left|V_0^-\right| \cos(\omega t + \beta z + \phi^-) e^{\alpha z}$$

Lossless transmission Line ($\alpha = 0$)

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$



$$Z_L = \frac{V(z=0)}{I(z=0)} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0$$

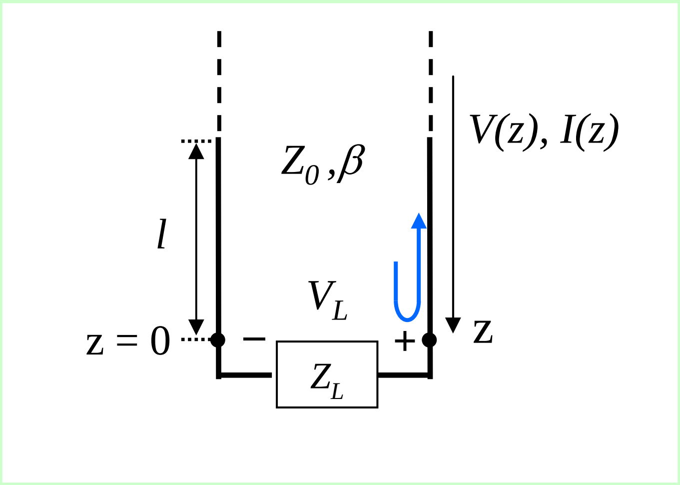
$$\Rightarrow \quad V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\therefore \quad \Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Soft-Matter Optical Spectroscopy

Lossless transmission Line ($\alpha = 0$)

$$\Gamma(z=l) = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \Gamma(0) e^{-2j\beta l}$$



$$Z_{in}(z=-l) = \frac{V(-l)}{I(-l)} = \frac{V_0^+\left[e^{j\beta l} + \Gamma e^{-j\beta l}\right]}{V_0^+\left[e^{j\beta l} - \Gamma e^{-j\beta l}\right]} Z_0 = \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$