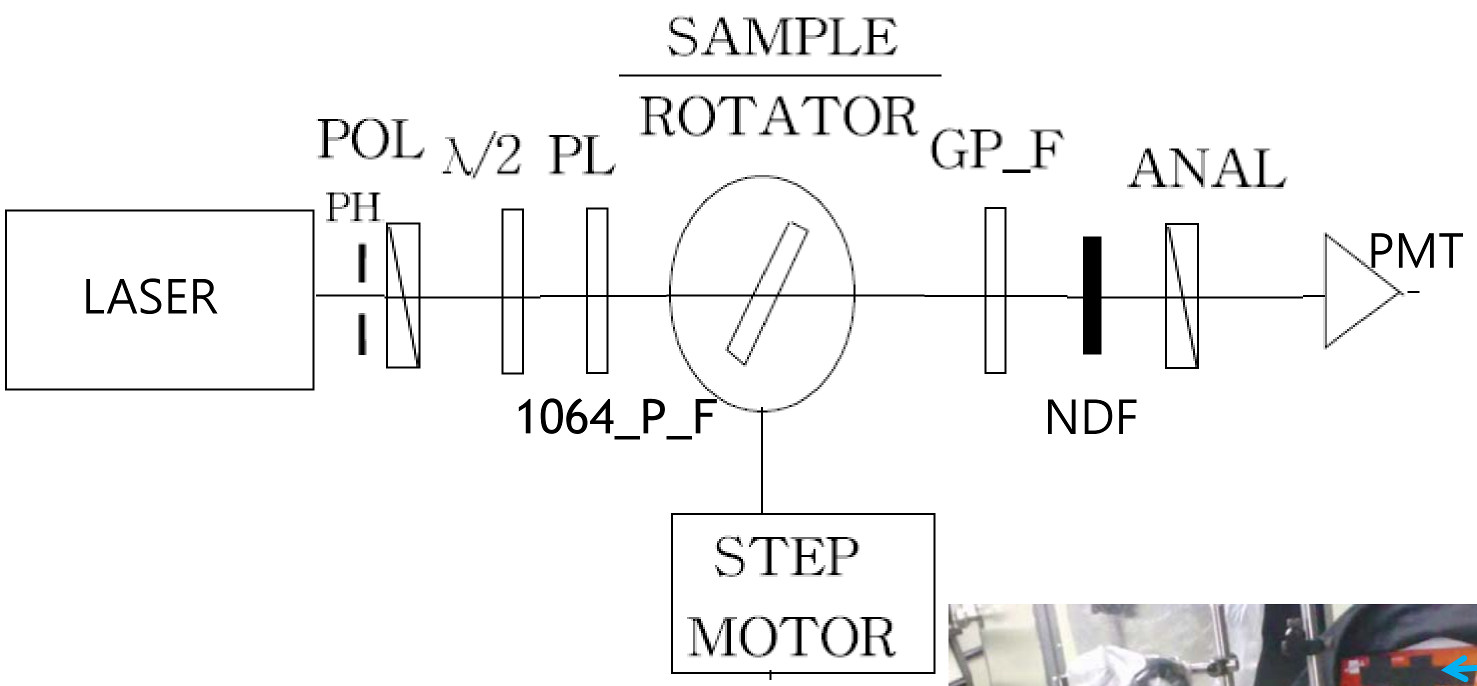
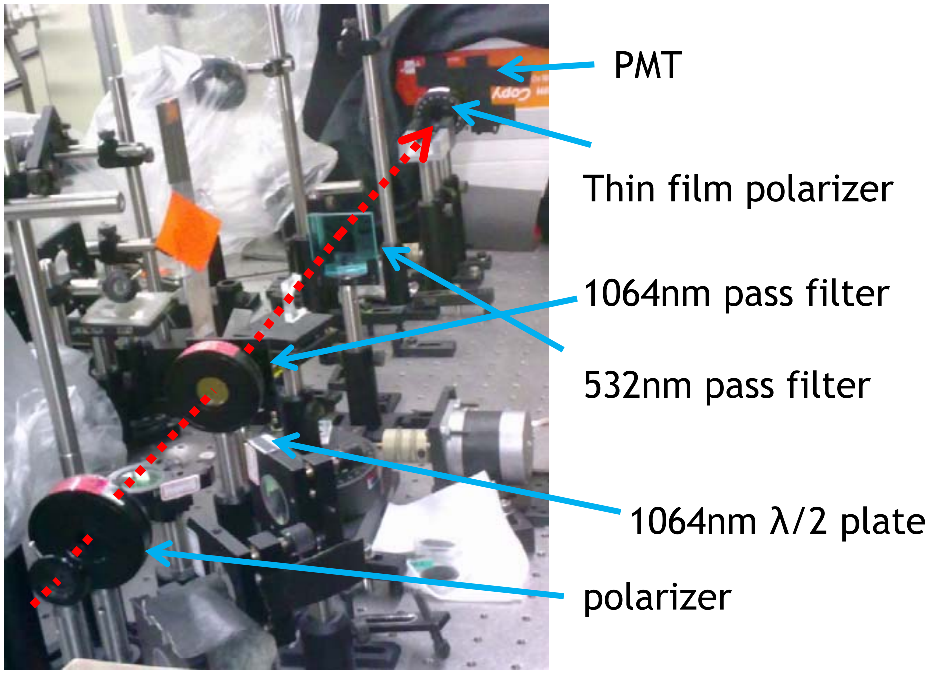
# The Measurements of the 2nd Nonlinear Optical Coefficients through Maker Fringe Experiments

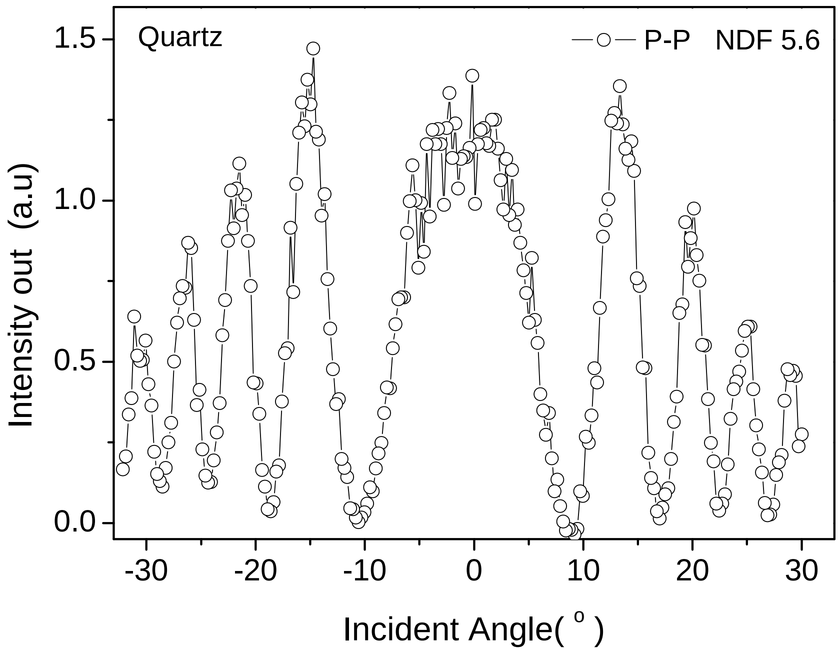
# Maker fringe Setup



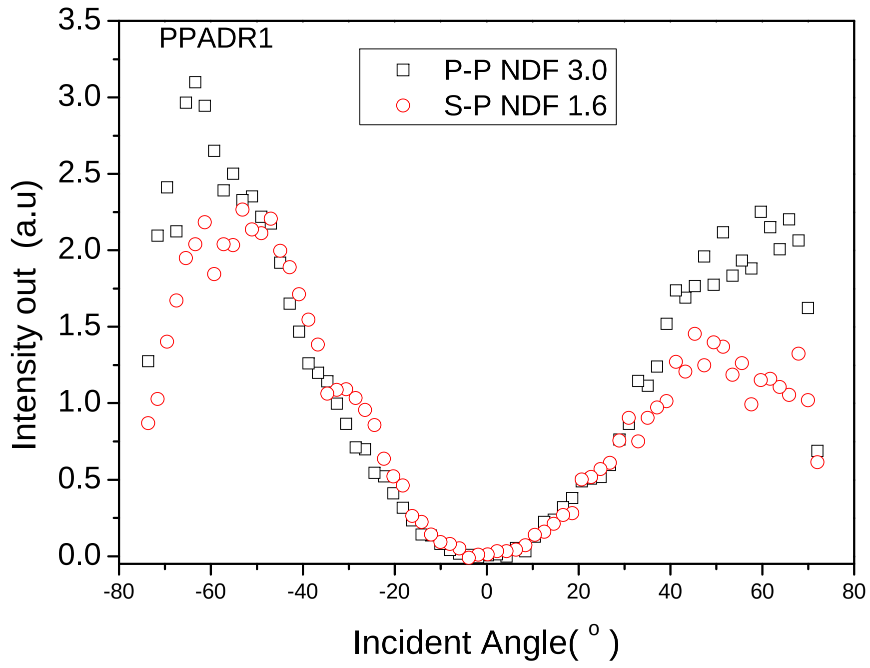



# Result

Y-cut Quartz



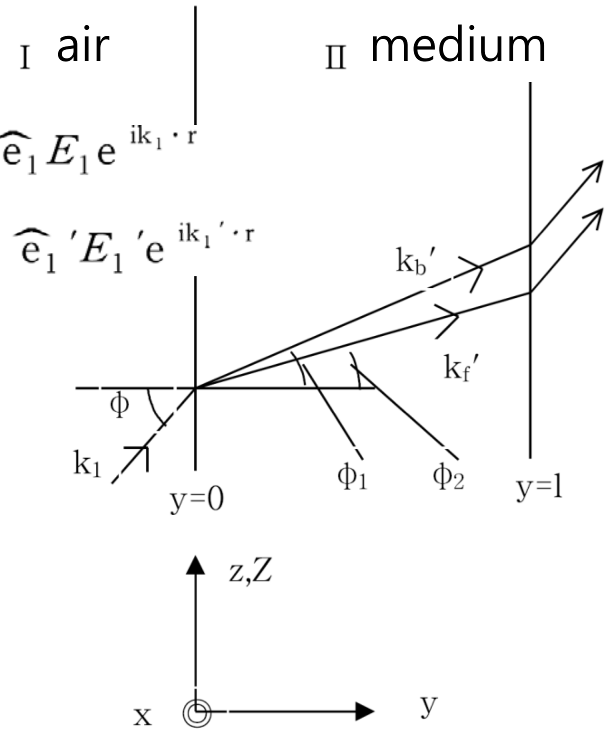

Polymer



# Theory

Incdent wave : $\mathrm{E}_1(\mathrm{r}) = \hat{\mathrm{e}}_1 E_1 \mathrm{e}^{i\mathrm{k}_1 \cdot \mathrm{r}}$

Refraction wave : $\mathrm{E}_1'(\mathrm{r}) = \hat{\mathrm{e}}_1' E_1' \mathrm{e}^{i\mathrm{k}_1' \cdot \mathrm{r}}$




$$\mathrm{E}_2''(\mathrm{r}) = \hat{\mathrm{e}}_2 E_2'' e^{i\mathrm{k}_2 \cdot \mathrm{r}}$$

$$\mathrm{H}_2''(\mathrm{r}) = \frac{c}{2\omega} \mathrm{k}_2 \times \hat{\mathrm{e}}_2 E_2'' e^{i\mathrm{k}_2 \cdot \mathrm{r}}$$

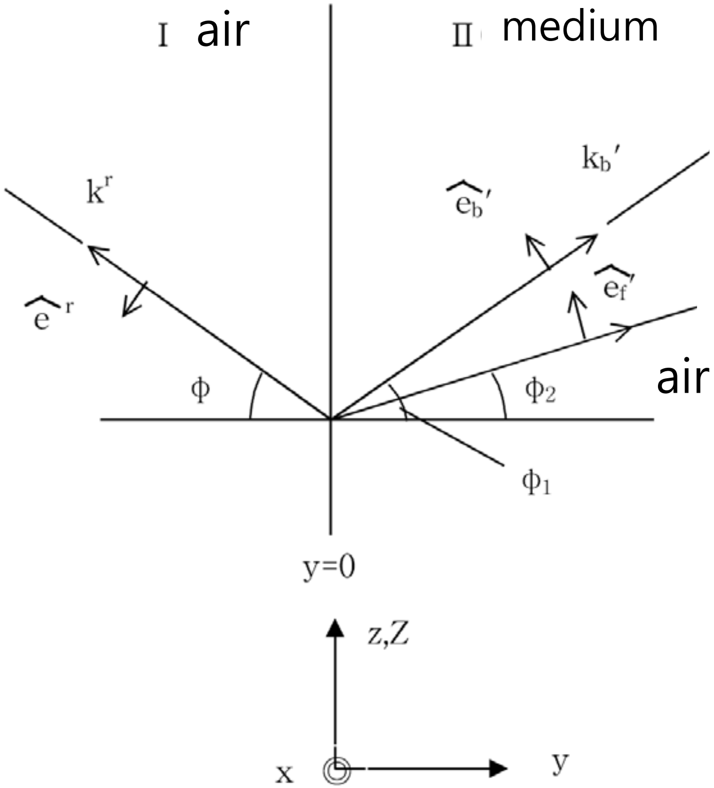
$$\mathrm{E}_2'(\mathrm{r}) = \hat{\mathrm{e}}_f' E_{2f}' e^{i\mathrm{k}_f' \cdot \mathrm{r}} + \hat{\mathrm{e}}_b' E_{2b}' e^{i\mathrm{k}_b' \cdot \mathrm{r}}$$

$$\mathrm{H}_2'(\mathrm{r}) = \frac{c}{2\omega} \mathrm{k}_f' \times \hat{\mathrm{e}}_f' E_{2f}' e^{i\mathrm{k}_f' \cdot \mathrm{r}} + \frac{c}{2\omega} \mathrm{k}_b' \times \hat{\mathrm{e}}_b' E_{2b}' e^{i\mathrm{k}_b' \cdot \mathrm{r}}$$

$$; E_{2b}' \approx \frac{4\pi}{n_1(\phi_1)^2 - n_2(\phi_1)^2} P_{2,eff}^{(2)\prime}$$

$$P_{2,eff}^{(2)\prime} = \hat{\mathrm{e}}_b' \cdot \mathrm{P}_2^{(2)\prime}$$

# In boundary surface




$$\mathrm{I}: \quad E_2(r) = \hat{e}_r E_2^r \, e^{ik^r \cdot r}$$

$$H_2(r) = \frac{c}{2\omega} k^r \times \hat{e}_r E_2^r \, e^{ik^r \cdot r}$$

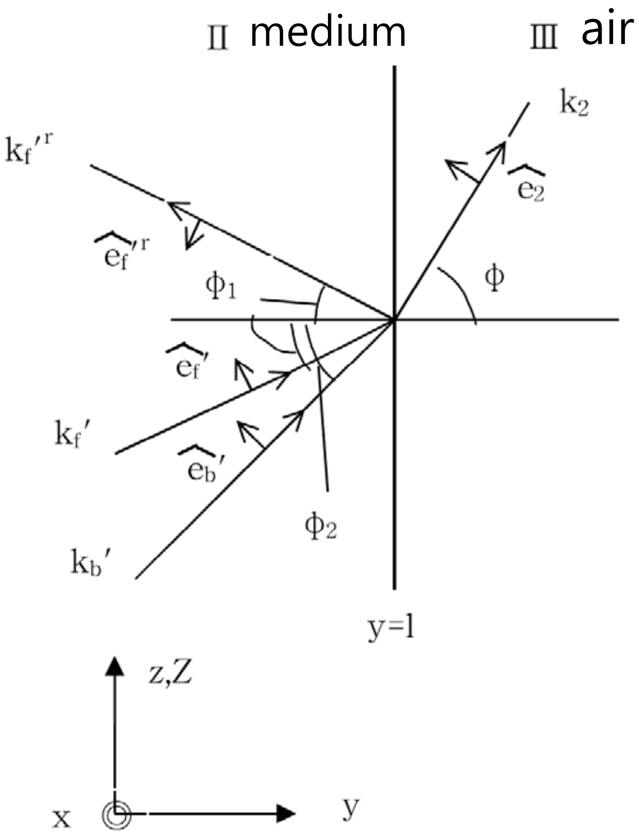
$$\mathrm{II}: \quad E_2'(r) = \hat{e}_f' E_{2f}' \, e^{ik_f' \cdot r} + \hat{e}_b' E_{2b}' \, e^{ik_b' \cdot r}$$

$$H_2'(r) = \frac{c}{2\omega} k_f' \times \hat{e}_f' E_{2f}' \, e^{ik_f' \cdot r} + \frac{c}{2\omega} k_b' \times \hat{e}_b' E_{2b}' \, e^{ik_b' \cdot r}$$

boundary condition

$$E_{2f}' = -\frac{\cos\phi_1 + n_1 \cos\phi}{\cos\phi_2 + n_2(\phi_2)\cos\phi} E_{2b}'$$

# In boundary surface




Ⅱ: $$E_2'(r) = \hat{e}_f' E_{2f}' e^{ik_f'\cdot r} + \hat{e}_b' E_{2b}' e^{ik_b'\cdot r} + \hat{e}_f'^r E_{2f}'^r e^{ik_f'^r\cdot r}$$

$$H_2'(r) = \frac{c}{2\omega} k_f' \times \hat{e}_f' E_{2f}' e^{ik_f'\cdot r} + \frac{c}{2\omega} k_b' \times \hat{e}_b' E_{2b}' e^{ik_b'\cdot r} + \frac{c}{2\omega} k_f'^r \times \hat{e}_f'^r E_{2f}'^r e^{ik_f'^r\cdot r}$$

Ⅲ: $$E_2''(r) = \hat{e}_2 E_2'' e^{ik_2\cdot r}$$

$$H_2''(r) = \frac{c}{2\omega} k_2 \times \hat{e}_2 E_2'' e^{ik_2\cdot r}$$

boundary condition

$$E_2'' e^{ik_2\cdot\hat{z}l} = \frac{n_2(\phi_2)\cos\phi_2}{n_2(\phi_2)\cos\phi + \cos\phi_2} E_{2f}' e^{ik_f'\cdot\hat{z}l} + \frac{n_2(\phi_2)\cos\phi_1 + n_1\cos\phi_2}{n_2(\phi_2)\cos\phi + \cos\phi_2} E_{2b}' e^{ik_b'\cdot\hat{z}l}$$

# Intensity of SHG

$$I_{2,\,out} = \frac{c}{8\pi}\ |E_2{}''|^2$$

$$I_{2,\,out} \approx \frac{8\pi c\,(P^{(2)}_{2,\,eff}{}')^2}{[n_1^2 - n_2(\phi_1)^2]^2} \times \frac{[2n_2(\phi_2)\cos\phi_2][n_2(\phi_2)\cos\phi_1 + n_1\cos\phi_2]}{[n_2(\phi_2)\cos\phi + \cos\phi_2]^3} \times [\cos\phi_1 + n_1\cos\phi]\sin^2\Psi$$

$$;\Psi = \frac{2\pi l}{\lambda}[n_1\cos\phi_1 - n_2(\phi_2)\cos\phi_2]$$

$$P^{(2)}_{2,\,eff}{}' = 2d_{eff}(E_1{}')^2$$

$$= 2d_{eff}[t^{(1)}_{as}]^2(E_1)^2 \qquad ;\ 2d_{eff} = X_{eff}$$

$$= \frac{16\pi}{c} d_{eff}[t^{(1)}_{as}]^2 I_{1,\,in} \qquad ;\ t^{(1)}_{as} =$$ Coefficient of transmission in Air & medium

$$I_{2,out} = \frac{2048\pi^3}{c} \frac{I^2_{1,in} d^2_{eff} [t^{(1)}_{as}]^4}{[n_1(\phi_1)^2 - n_2(\phi_1)^2]^2} T_2 \sin^2\Psi$$

$$;\Psi = \frac{2\pi l}{\lambda}[n_1 \cos\phi_1 - n_2(\phi_2)\cos\phi_2],$$

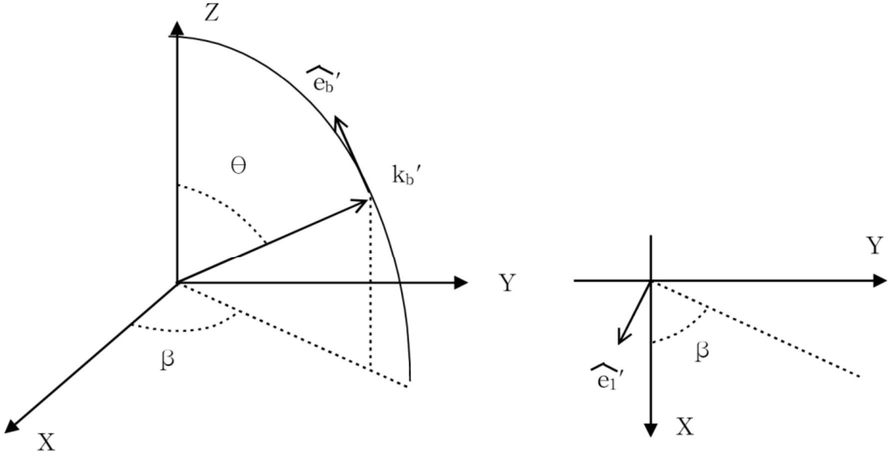
$$T_2 = \frac{[2n_2(\phi_2)\cos\phi_2][n_2(\phi_2)\cos\phi_1 + n_1\cos\phi_2]}{[n_2(\phi_2)\cos\phi + \cos\phi_2]^3} \times [\cos\phi_1 + n_1\cos\phi]$$

$$t^{(1)}_{as} = \begin{cases} \dfrac{2\cos\phi}{\cos\phi + n_1\cos\phi_1} & \text{(S-pol incident light)} \\ \dfrac{2\cos\phi}{n_1\cos\phi + \cos\phi_1} & \text{(P-pol incident light)} \end{cases}$$

$$n_1 = \begin{cases} n_{1o} & \text{(S-pol incident light)} \\ n_1(\phi_1) & \text{(P-pol incident light)} \end{cases}$$

# Obtain $d_{eff}$

S-pol incident light




electric field of fundamental wave :

$$E_{1X}' = E_1' \sin\beta,$$

$$E_{1Y}' = -E_1' \cos\beta,$$

$$E_{1Z}' = 0$$

Second order nonlinear polarization :

$$\begin{pmatrix} P_{2X}^{(2)\prime} \\ P_{2Y}^{(2)\prime} \\ P_{2Z}^{(2)\prime} \end{pmatrix} = 2\begin{pmatrix} d_{il} \end{pmatrix} \begin{pmatrix} E_{1X}'^2 \\ E_{1Y}'^2 \\ E_{1Z}'^2 \\ 2E_{1Y}'E_{1Z}' \\ 2E_{1Z}'E_{1X}' \\ 2E_{1X}'E_{1Y}' \end{pmatrix},$$

$$P_{2X}^{(2)\prime} = -4d_{22}E_{1X}'E_{1Y}' = 2d_{22}E_1'^2\sin2\beta,$$

$$P_{2Y}^{(2)\prime} = -2d_{22}[E_{1X}'^2 - E_{1Y}'^2] = 2d_{22}E_1'^2\cos2\beta,$$

$$P_{2Z}^{(2)\prime} = 2d_{31}(E_{1X}'^2 + E_{1Y}'^2) = 2d_{31}E_1'^2,$$

$$\begin{pmatrix} d_{il} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow P^{(2)}_{2,eff}{}' = \hat{e}_b{}' \cdot P^{(2)}_2{}'$$

$$= -P^{(2)}_{2X}{}' \cos\Theta \cos\beta - P^{(2)}_{2Y}{}' \cos\Theta \sin\beta + P^{(2)}_{2Z}{}' \sin\Theta$$

$$= 2E_1{}'^2 [d_{31} \sin\Theta - d_{22} \cos\Theta \sin 3\beta]$$

$$\Downarrow \quad P^{(2)}_{2,eff}{}' = 2\, d_{eff}\, E_1{}'^2$$

$$d_{eff} = d_{31} \sin\Theta - d_{22} \cos\Theta \sin 3\beta$$

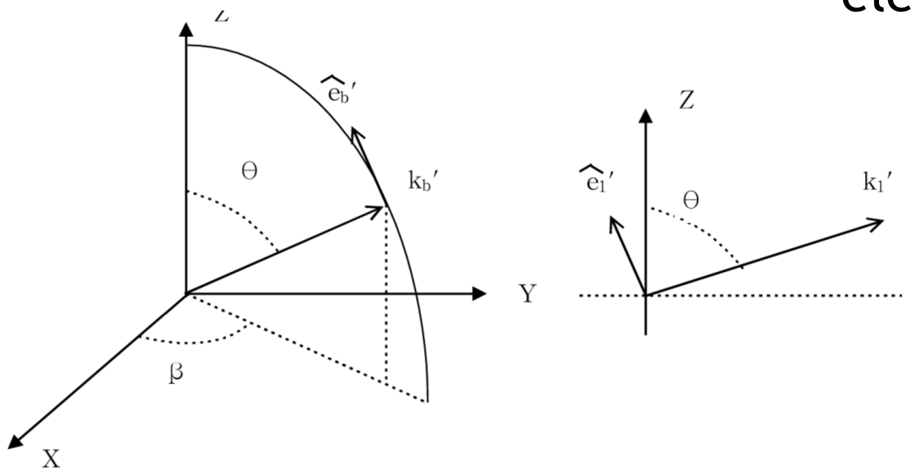
$$\Downarrow \quad \Theta = \frac{\pi}{2} - \phi_1$$

$$\boxed{d_{eff} = d_{31} \cos\phi_1 - d_{22} \sin\phi_1 \sin 3\beta}$$

$$\Downarrow \quad \beta = 90^\circ$$

$$\boxed{d_{eff} = d_{31} \cos\phi_1 + d_{22} \sin\phi_1}$$

## P-pol incident light




electric field of fundamental wave :

$$E_{1X}' = -E_1' \cos\Theta \cos\beta,$$

$$E_{1Y}' = -E_1' \cos\Theta \sin\beta,$$

$$E_{1Z}' = E_1' \sin\Theta$$

Second order nonlinear polarization :

$$P_{2X}^{(2)}{}' = 4[d_{31} E_{1Z}' E_{1X}' - d_{22} E_{1X}' E_{1Y}']$$

$$= -2E_1'^2 [d_{31} \sin 2\Theta \cos\beta + d_{22} \cos^2\Theta \sin 2\beta]$$

$$P_{2Y}^{(2)}{}' = 2[d_{22}(E_{1Y}'^2 - E_{1X}'^2) + 2d_{31} E_{1Y}' E_{1Z}']$$

$$= -2E_1'^2 [d_{22} \cos^2\Theta \cos 2\beta + d_{31} \sin 2\Theta \sin\beta],$$

$$P_{2Z}^{(2)}{}' = 2[d_{31}(E_{1X}'^2 + E_{1Y}'^2) + d_{33} E_{1Z}'^2]$$

$$= 2E_1'^2 [d_{31} \cos^2\Theta + d_{33} \sin^2\Theta]$$

$$P^{(2)}_{2,eff}{}' = \hat{e}_b{}' \cdot P^{(2)}_{2}{}'$$

$$= P^{(2)}_{2X}{}'\cos\Theta\cos\beta - P^{(2)}_{2Y}{}'\cos\Theta\sin\beta + P^{(2)}_{2Z}{}'\sin\Theta$$

$$= 2E_1{}'^2[d_{22}\cos^3\Theta\sin 3\beta + 3d_{31}\sin\Theta\cos^2\Theta + d_{33}\sin^3\Theta]$$

$$P^{(2)}_{2,eff} = 2\, d_{eff}\, E_1^2$$

$$d_{eff} = d_{22}\cos^3\Theta\sin 3\beta + 3d_{31}\sin\Theta\cos^2\Theta + d_{33}\sin^3\Theta$$

$$\Theta = \frac{\pi}{2} - \phi_1$$

$$d_{eff} = d_{22}\sin^3\phi_1\sin 3\beta + 3d_{31}\cos\phi_1\sin^2\phi_1 + d_{33}\cos^3\phi_1$$

$$d_{eff} = -d_{22}\sin^3\phi_1 + 3d_{31}\cos\phi_1\sin^2\phi_1 + d_{33}\cos^3\phi_1$$