Stress and Fold Location in Thin Elastic Membranes

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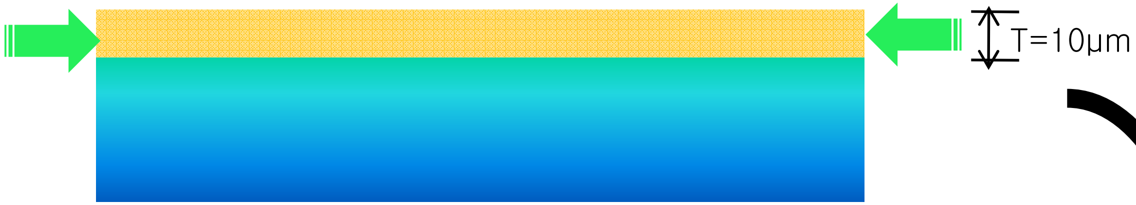
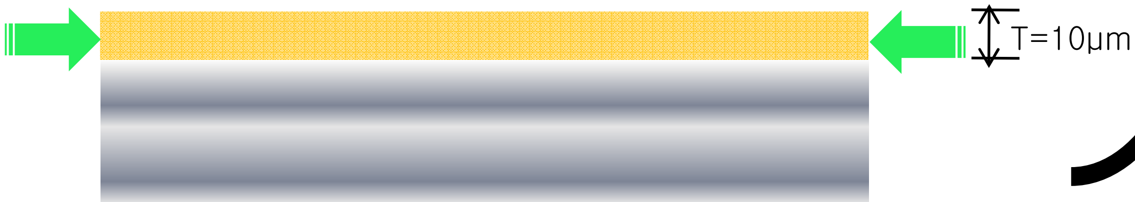
Thin layer / Various subphase

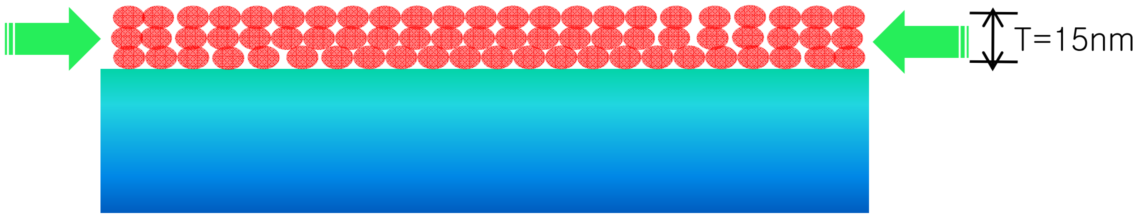


- (1) Polyester thin film on water -



- (2) Polyester thin film on Gel -

Model of biological membrane
(ex: lung surface)



- (3) Gold nano particle on water (trilayer) -

Nano particle layer having unique electronic, optical and magnetic properties

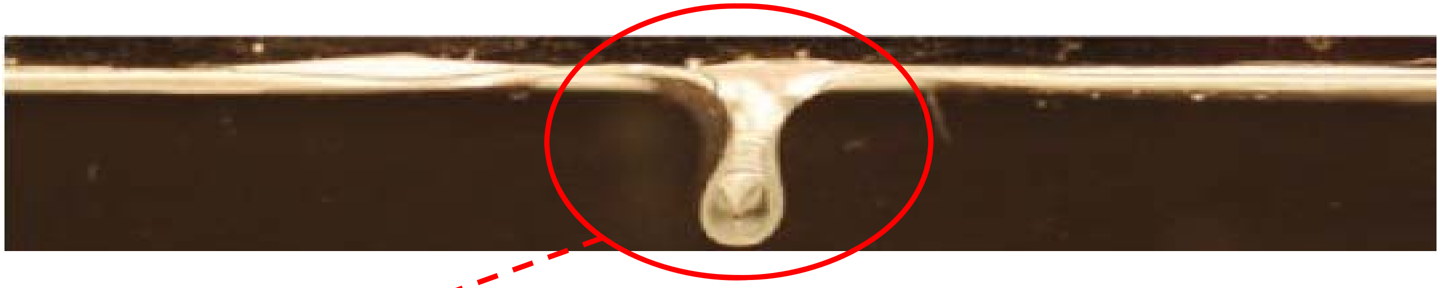
Observation By Using Microscope

- (1) Polyester thin film on water (side view) -

λ = 1.6cm

- Adjust low pressure: Sinusoidal wrinkling occurred

- Higher pressure:A large wrinkle appeared at the center

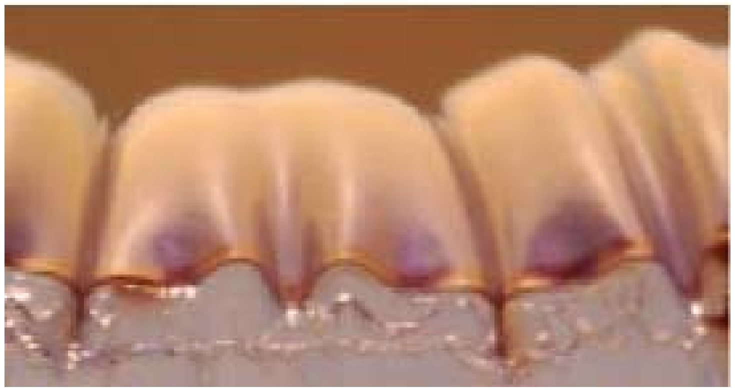


- Over critical pressure: Sinusoidal wrinkles were relaxed and a folding appeared at the center

Observation By Using Microscope

- (2) Polyester thin film on gel (side view) -

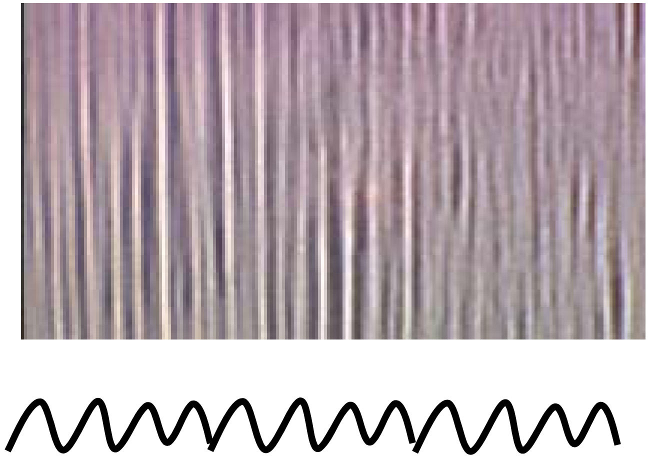
λ = 3.3mm

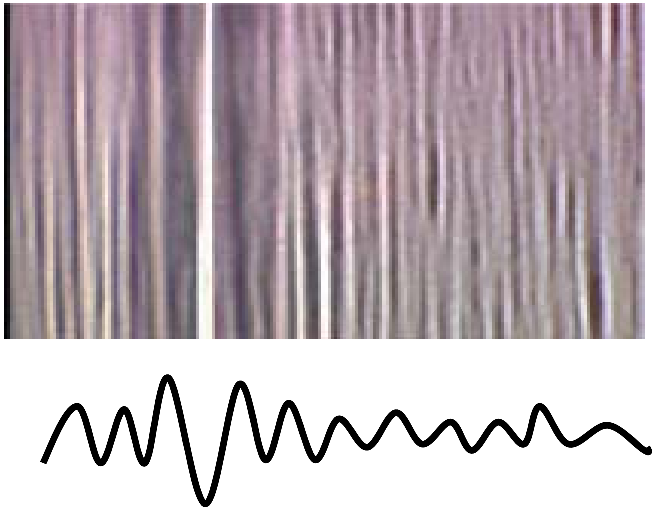


-It Shows the same result as the case of Polyester /water interface but the period of wrinkle was contracted to 3.3mm (substrate difference)

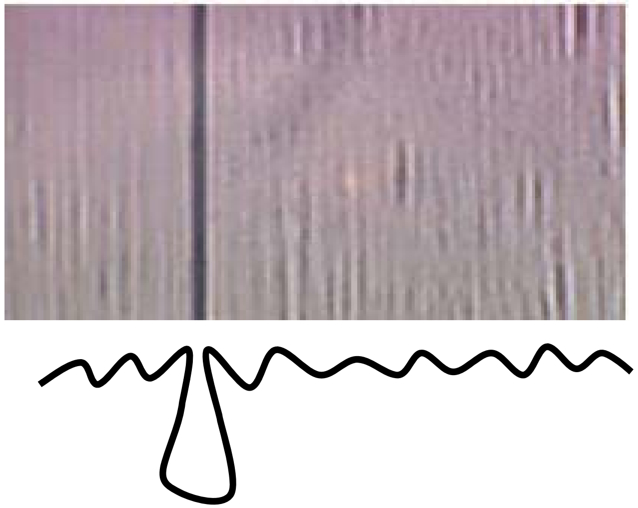
Observation By Using Microscope

- (3) Gold nano particle on water (top view) - $\lambda \approx 10\mu m$





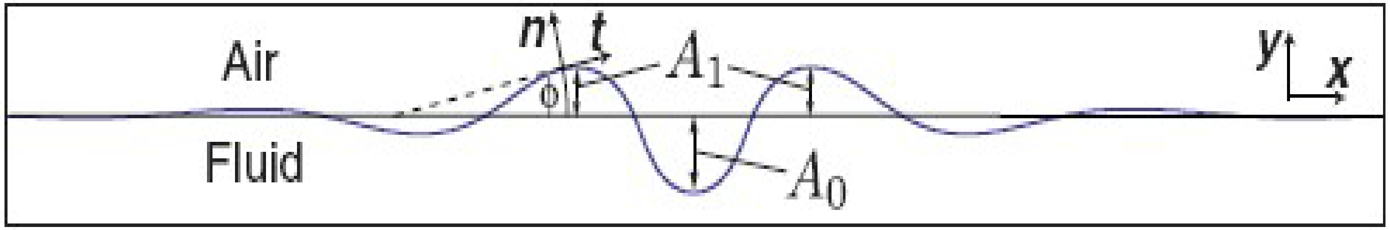
When the wrinkle amplitude at the center grow maximum, there is a bright stripe due to light scattering



But after wrinkle to folding transition there is a dark stripe due to enclosing folding

Interpretation

System of the surface moves to minimize the surface energy !!



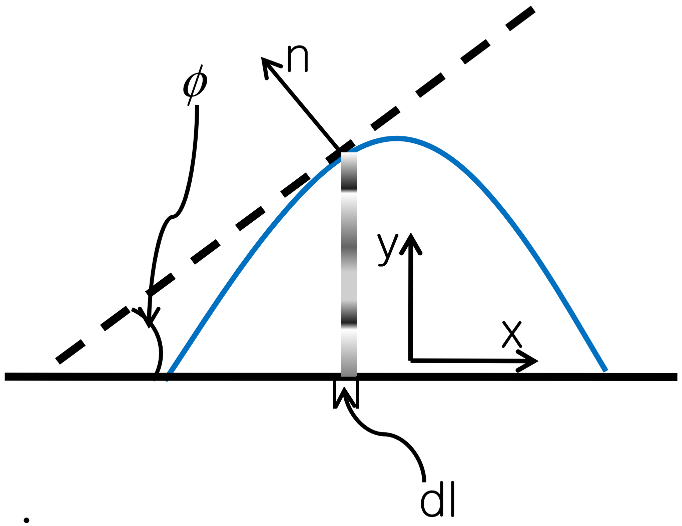
Total Energy: Bending energy of the film
+ gravitational energy of the subphase

Interpretation

$$U = U_B + U_K = \frac{B}{2}\int_0^L dl(\dot{\phi})^2 + \frac{K}{2}\int_0^L dl\cos\phi\, y^2$$

$\frac{B}{2}\int_0^L dl(\dot{\phi})^2$: Bending energy (similar to the energy of spring)

$\frac{K}{2}\int_0^L dl\cos\phi\, y^2$: Gravitational energy



$$* \int_0^L \int_0^y \rho g y' (dl\cos\phi) dy' = \frac{\rho g}{2}\int_0^L dl\cos\phi\, y^2$$

ϕ : angle between tagent of curve and x-axis

$\dot{\phi}$: curvature (derivative of ϕ with arc length)

B: Bending stiffness, K=ρg

Interpretation

Constraint: the projection length is same as the total length of the film - deformation length, Δ

$$\text{constraint} : L - \Delta = \int_0^L dl \cos\phi$$

$$L - \Delta = \int_0^L dl \cos\phi = \int_0^L dl - \Delta$$

$$\rightarrow \int_0^L dl(1 - \cos\phi) - \Delta = \int_0^L dl(1 - (1 - \frac{\phi^2}{2!} + \ldots)) - \Delta \simeq \int_0^L dl(\frac{(\dot{y})^2}{2} - \frac{\Delta}{L}) = 0$$

Effective Lagrangian (Lagrangian+constraint) is,

$$L^* = U_B + U_K + \beta \int_0^L dl(\frac{(\dot{y})^2}{2} - \frac{\Delta}{L})$$

Interpretation

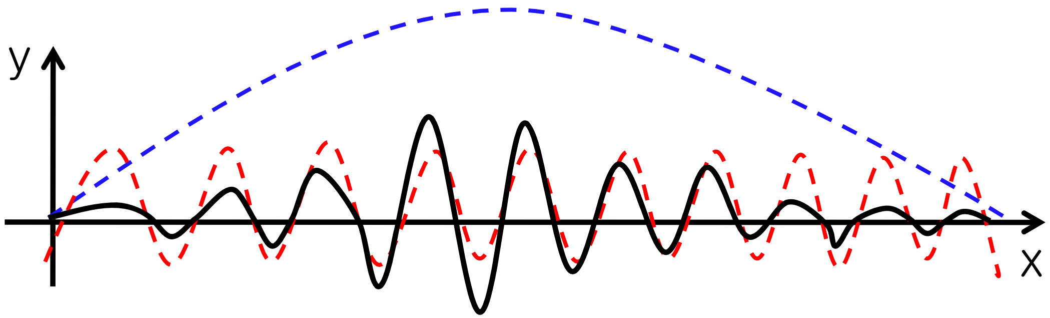
Minimizing the Lagrangian yields the equivalent to the first Foppl-von Karman equations

$$By^{(4)} + \beta y^{(2)} + Ky = 0$$

The solution that satisfies the boundary condition,

$$y(0) = \dot{y}(0) = y(L) = \dot{y}(L) = 0$$

$$y(\ell) = A\sin(\pi\ell/L)\sin(2\pi\ell/\lambda) \qquad \lambda = 2\pi(B/K)^{1/4}$$



Interpretation

(1) $\lambda = 2\pi (B/K)^{1/4}$: The wavelength of the wrinkle when small amount of deformation occurs depend only on the property of film and substrate not the size of the system

(2) $L - \Delta = \int_0^L dl \cos\phi = \int_0^L dl - \Delta$

$$\Delta = \int_0^L dl (1 - \cos\phi) \simeq \frac{1}{2} \int_0^L dl (\dot{y})^2 \propto L (A/\lambda)^2$$

$A \propto \lambda \sqrt{(\Delta / L)}$ → **Amplitude also depend on deformation ratio (dimensionless)**

Interpretation

(3) Total energy is approximately,

$$U = U_B + U_K = \frac{B}{2}\int_0^L dl(\dot{\phi})^2 + \frac{K}{2}\int_0^L dl \cos\phi y^2 \propto BL(A/\lambda^2)^2 + KLA^2$$

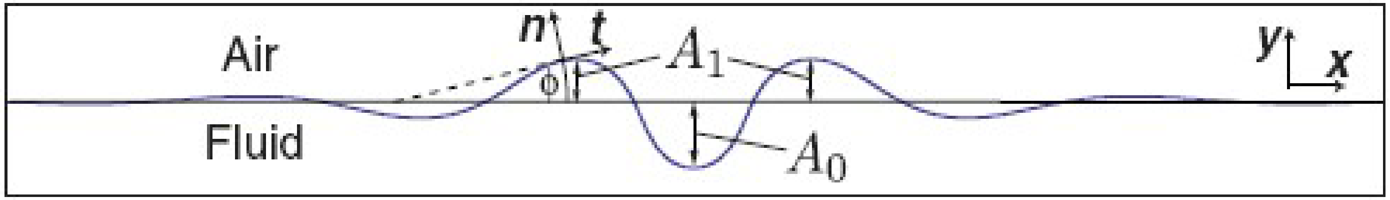
$$\propto (BK)^{1/2}\Delta$$

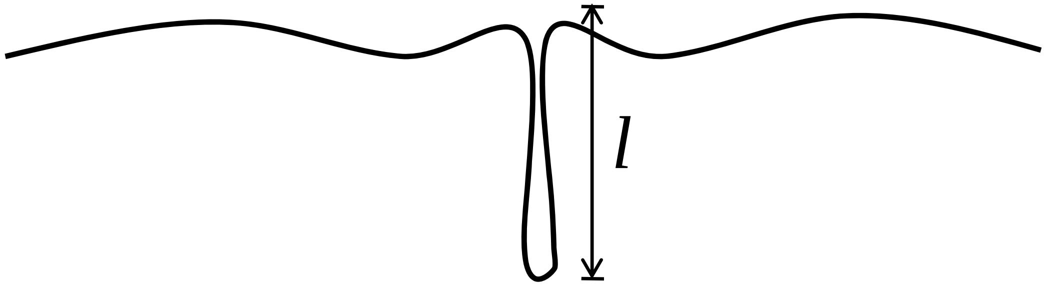
So specific energy also independent of the system size.

$$u = U/L \propto (BK)^{1/2}$$

Wrinkle to Folding Transition

- When the Folding occurs, another amplitudes are relaxed except A_0





- $$U_B \simeq \left(\frac{B}{2}\right)\int_0^L dl(\dot{\phi}_{\max})^2 \propto B\dot{\phi}_{\max} = B/l$$

$$U_K = \left(\frac{K}{2}\right)\int_0^L dl \cos\phi\, y^2$$

$$= \left(\frac{K}{2}\right)\int_0^L dl y^2 - \left(\frac{K}{2}\right)\int_0^L dl(1-\cos\phi)y^2 \propto Kl\Delta^2 - K\Delta^3$$

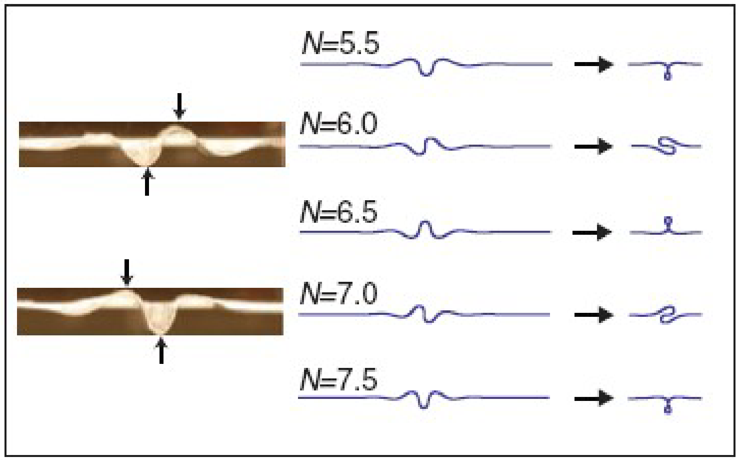
- l Is determined by minimization of total energy

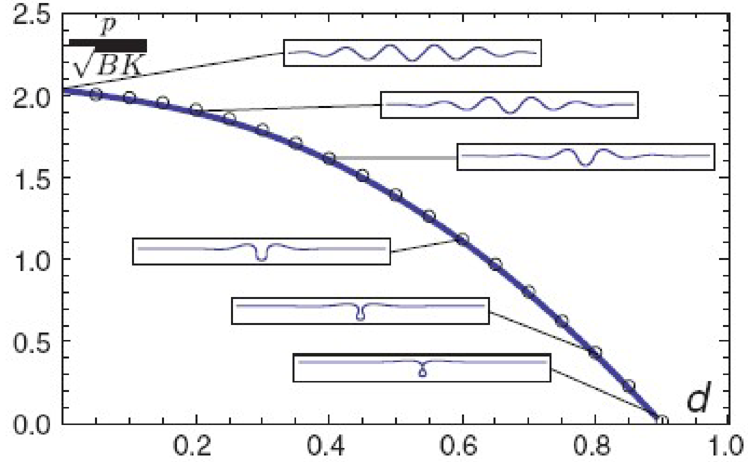
Wrinkle to Folding Transition

$$\partial_l (U_B + U_K) = 0 \rightarrow l = (B/K)^{1/2}(1/\Delta)$$

$$U_{\min} \propto (BK)^{1/2}\Delta - K\Delta^3$$

$$p = \partial_\Delta U_{\min} \propto (BK)^{1/2} - K\Delta^2 = a - bd^2$$



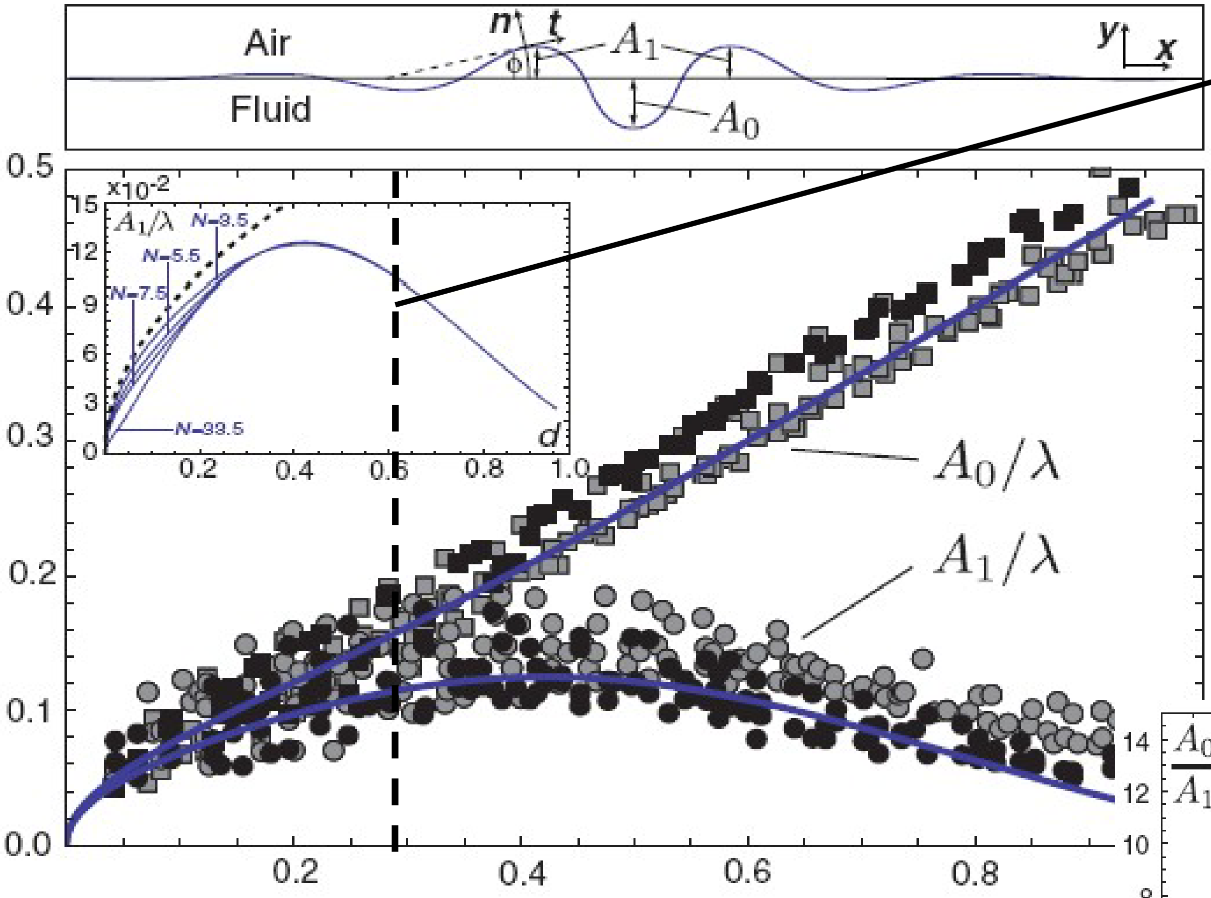


Left : d=0.5 → Right: d=0.9

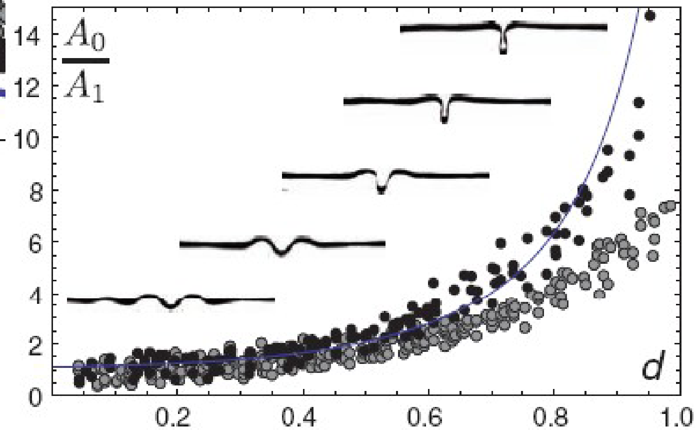
$$d = \Delta/\lambda,\ N = L/\lambda$$

(dimensionless parameters to describe deformation and system scale)

Wrinkle to Folding Transition



d>0.3 : A_0 increases linearly
A_1 has maximum and decrease



Summary

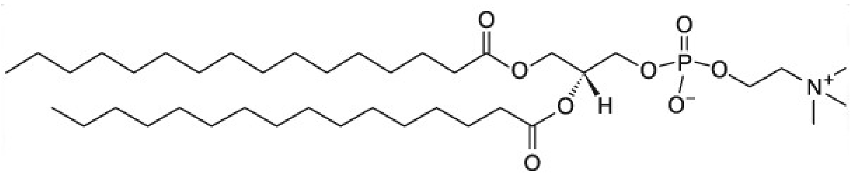
- The wrinkle to folding transition can be observed by light microscope and interpreted by bending energy of layer and gravitational potential energy of substrate

- The amplitude and wavelength of wrinkles are determined by properties of film and substrate

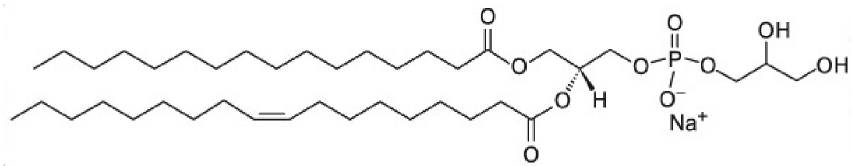
- To minimize the total energy, folding is formed in high deformation condition

Furthermore

Collapse of the lipid monolayer :



DPPC(1,2-dihexadecanoyl-*sn*-glycero-3-phosphocholine)



POPG(1-palmitoyl-2-oleoyl-sn-glycero-3-phospho-(1'-rac-glycerol) PG

It is related to our recent work to investigate the polar lipid/water interface